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Heaps of Segments and Lorentzian Quantum Gravity

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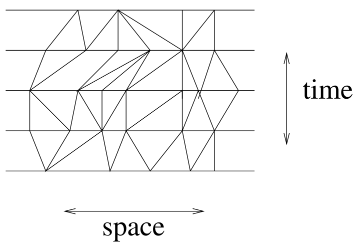
Summary by Sylvie Corteel

Abstract

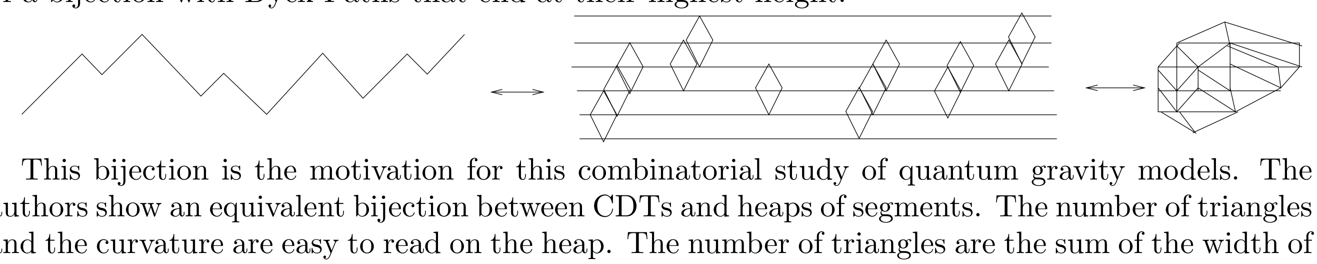
This work is a combinatorial study of quantum gravity models related to Lorentzian quantum gravity. These models are discrete combinatorial objects called dynamical triangulations. They are related to classical combinatorial objects: Dyck paths, heaps, ... This work is in collaboration with Xavier Viennot and is part of the speaker's thesis [4].

Quantum gravity is a quantum description of the space-time geometry. We refer to Loll [5] for precise definitions. In the Lorentzian quantum gravity the universe can be of dimension (1+1) [1]. One dimension is for space, the other one for time. This defines the dynamical triangulations.

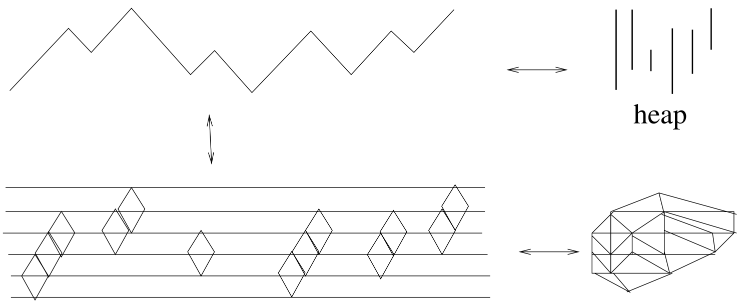


Moreover space is supposed to be circular and to have a unique origin. This gives some special cylindric dynamical triangulations (CDT). These CDT can be cut and made planar by choosing a first vertex following its rightmost link and cutting. These gives dynamical triangulations with the condition that all the rightmost triangles points up.

The problem is therefore to enumerate these triangulations according to different parameters, for example according to the number of triangles. The model was solved by Di Francesco, Guitter and Kirstjansen [3] and they included a parameter that measured the absolute value of the local curvature. They used transfer matrix methods to solve the model, which they presented in terms of a bijection with Dyck Paths that end at their highest height.



This bijection is the motivation for this combinatorial study of quantum gravity models. The authors show an equivalent bijection between CDTs and heaps of segments. The number of triangles and the curvature are easy to read on the heap. The number of triangles are the sum of the width of the segments and the curvature is twice the number of segments minus the number of contacts. We refer to [4] for detailed definitions. Then using the heap machinery [7, 6] multi-variate generating functions can be deduced.



This gives a series of solutions of existing models and show how different restrictions to the topology of the universes can be modelled by similar classes of heaps. In particular connected heaps [2] appear naturally. This work and extensions can be found in the speaker's thesis [4].

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