Asymptotic Bounds for the Fluid Queue Fed by Subexponential on/off Sources

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[summary by Jean-Marc Lasgouttes]

This talk presents results from Dumas and Simonian [3] on the tail behaviour of the buffer content of a fluid queue processing the input of several exponential and subexponential sources. While the results in [3] are rather general, the presentation given here uses a simplified setting, for the sake of understandability.

1. Framework

Consider a fluid queue with infinite buffering capacity and outflow rate c. This queue is fed by $N > 1$ independent stationary on/off sources, where source i, $1 \le i \le N$ is characterized by:

- silence periods, where it generates no traffic, of length S_{in}, $n \ge 1$, i.i.d. and exponentially distributed;
- activity periods, where it generates traffic at peak rate h_i, of length A_{in}, $n \ge 1$; these variables are i.i.d., but no assumption is made on their distribution for now.

The following notation will be useful later:

$$p_i := \frac{\mathbb{E}[A_{in}]}{\mathbb{E}[A_{in} + S_{in}]}, \quad \rho_i := h_i p_i.$$

To characterize the stationary regime of source i, it is convenient to introduce the time elapsed in the current activity period A_i^*, whose distribution is given by

$$\Pr[A_i^* = 0] = 1 - p_i,$$

$$\Pr[A_i^* > x | A_i^* > 0] = \int_x^\infty \frac{\Pr[A_{in} > y]}{\mathbb{E}[A_{in}]} dy.$$

In what follows, we restrict ourselves to the case where $h_i \equiv h$, $p_i \equiv p$ and $\rho_i \equiv \rho$, for all $1 \le i \le N$. If V_t is the volume of fluid in the buffer at time t (with $V_0 = 0$), then the following result is well known:

Theorem 1. *Let $\Omega_i[t]$ be the flow emitted by source i in stationary regime in the interval $]-t, 0]$ and define $\Omega[t] := \sum_{i=1}^N \Omega_i[t]$. Then, assuming $N\rho < C$,*

$$\lim_{t \to \infty} V_t \overset{\mathcal{L}}{=} V := \sup_{t \ge 0} (\Omega[t] - ct).$$

It is important to have good estimates for $\Pr[V > x]$, since this can be used to determine loss rate in a finite buffer queue. A typical result in this respect is due to Anick, Mitra and Sondhi [1]: if there exist constants α_i such that $\Pr[A_{in} > x] = O(e^{-\alpha_i x}), 1 \le i \le N$, then there exists α such that $\Pr[V > x] = O(e^{-\alpha x})$.

However, recent studies have shown that some sources may have subexponential activity patterns, such as $\Pr[A_{in} > x] = O(x^{-s_i})$, $s_i > 1$. The purpose of this work is therefore to find good estimates for the tail distribution of V when the sources are a mix of exponential and subexponential sources, extending the results of [2, 4, 5].

2. Lower and Upper Bounds

Let I be a subset of $\{1, \ldots, N\}$, with cardinal $|I|$, and define

$$A_I^* := \min_{i \in I} A_i^*, \qquad \Omega_{\bar{I}}[t] := \sum_{i \notin I} \Omega_i[t],$$
$$n_0 := \inf\{n \geq 0 | nh + (N - n)\rho > c\}.$$

Then the following bound holds as $x \to \infty$:

$$\begin{aligned}\Pr[V > x] &\geq \max_I \Pr\left[(|I|h + (N - |I|)\rho - c)A_I^* > x\right] \\ &\geq \max_{|I| = n_0} \prod_{i \in I} \Pr\left[(n_0 h + (N - n_0)\rho - c)A_i^* > x\right].\end{aligned}$$

Similarly, defining V_i as

$$V_i := \sup_{t \geq 0} \left(\Omega_i[t] - \rho(1 + \epsilon)t\right),$$

where $\epsilon > 0$ is such that $(n_0 - 1)h + (N - n_0 + 1)\rho(1 + \epsilon) = c$, one has

$$\Pr[V > x] \leq \sum_{|I| = n_0} \prod_{i \in I} \Pr\left[V_i > \frac{x}{N - n_0 + 1}\right].$$

3. Application to a Mix of Exponential and Subexponential Sources

Assume that the queues can be partitioned in two classes for some $N_0 < N$:

$$\Pr[A_{in} > x] = O(x^{-s}), \qquad 1 \leq i \leq N_0,$$
$$\Pr[A_{in} > x] = O(e^{-\alpha_i x}), \qquad N_0 < i \leq N.$$

Then the main result of this study is as follows.

Theorem 2. *The following approximations hold:*
- *if* $N_0 < n_0$, *then* $\Pr[V > x] = O(e^{-\alpha x})$;
- *if* $N_0 \geq n_0$, *then* $\Pr[V > x] = O(x^{-n_0(s-1)})$.

Bibliography

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