Algorithmic Problems in Non-Cabled Networks

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[summary by Xavier Gourdon]

Abstract

Due to the specific nature of radio networks, the channel access in radio local area networks (LAN) is different from cabled LAN. The HIPERLAN standard for radio networks will provide a 24Mbps data rate transmission. It has a special feature, called active signalling, which can be used to provide an efficient channel access mechanism.

1. Active signalling

The channel access in radio LANs has to face special problems. Unlike wired LANs, nodes cannot build a complete history of the network from the fragments of feedback they obtain from the channel. This feature makes the collision detection techniques in radio networks different from cabled networks.

The active signalling feature of the radio LAN standard HIPERLAN provides an efficient channel access mechanism. It consists in requiring each node that wants to access the channel to send a certain sequence of on/off's as a preamble to each packet transmission. This sequence is encoded according to a random pattern whose details will be described later. The objective is to use these patterns to select (with a high probability) only one node so that no collision occurs during packet transmission. The patterns are also functions of the access priority assigned to the packet. During the transmission of its pattern and when it is in the "off" period, the node senses the channel: if it detects any other signal, then the node stops its pattern transmission and defers until the next attempt.

2. HIPERLAN active signalling pattern selection

The HIPERLAN active signalling pattern is divided into two consecutive phases, the access priority assertion phase and the contention phase.

2.1. The access priority assertion phase. The first slots of the pattern are dedicated to priority signalling.

The priority phase consists in leaving a certain number of idle (off) slots before one busy (on) slot. This number of slots is equal to 5 (maximum number of priority levels) minus the priority level. The priority assertion phase ends with the first busy slot encountered, called the priority pulse. Therefore, only the contenders with the highest access priority level survive to the priority assertion phase. Figure 1 shows an example where node B, on access priority level 3, beats node A on access priority level 2.

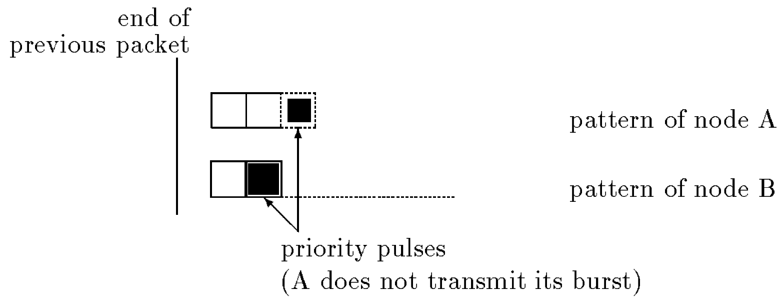

FIGURE 1. How to win priority contention

2.2. The contention phase. The contention phase is divided into two consecutive parts, the elimination part and the tail selection part. The role of the elimination part is to select a small number of survivors from a large number of contenders. The tail selection part tries to select only one survivor from a small number of contenders with a high probability. If more than one survivor is selected at the end of the contention phase, a collision occurs.

The elimination phase. It consists in enlarging the priority pulse with a random number of slots. Each node stretches its pulse independently of the other nodes and according to a geometric distribution of probability $p = 1/2$. Therefore the pulse is larger than k slots with probability $1/2^k$.

After the stretched pulses, the node leaves an idle slot, called the *survival verification slot* where it senses the channel. Only the contenders which simultaneously hold the highest access priority level and select the longest stretched pulse survive to the elimination part. Figure 2 shows an example where node A with stretch length of 1 slot is eliminated by node B with stretch length of 2 slots.

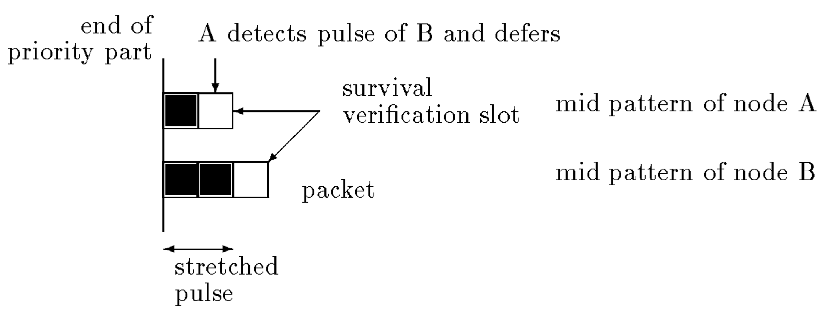

FIGURE 2. How to win the elimination part

The tail selection phase. This phase is also called the "yield" part and follows just after the survival verification slot. The nodes which survived to the elimination part again select a random number of slots according to a geometric rate $1 - r$ with $r = 1/8$. But instead of transmitting busy slots again as with the stretched pulse procedure, the contenders terminate their pattern with a number of idle slots equal to their respective new selected numbers. Thus if a node detects no signal during its silent period, then the node transmits its packet. Otherwise, the node defers until the next access cycle.

Therefore, only the nodes whose patterns simultaneously present the highest access priority level, the longest stretched pulse and the shortest "yield" part gain the right to transmit their packet.

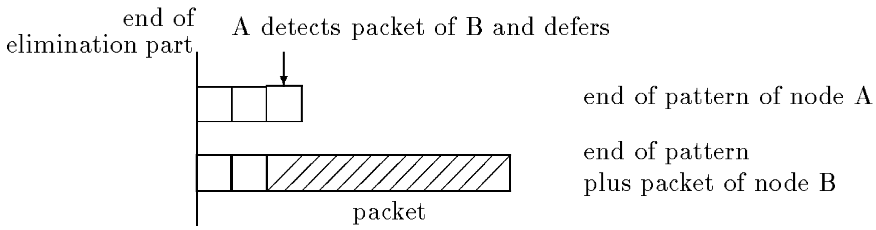

FIGURE 3. How to win the tail selection part

Figure 3 shows an example where node B with yield length of 2 slots is selected before node A with yield length of 3 slots.

3. Performance evaluation of HIPERLAN active signalling channel access

The contention phase has a certain length, called overhead. Starting with n contenders at the same highest priority level, we prove that the average contention overhead is $\log_2 n + O(1)$. Also, whatever the number of contenders, the residual collision rate on packet transmission is less than 3.5%.

3.1. Analytic evaluation of the elimination phase. Given n contenders at the highest priority level, we denote by S_n the average number of survivors after the elimination phase, L_n the average stretched pulse length of these survivors, and p_n the probability of having a single survivor.

With p the geometric stretching probability and $q = 1 - p$, the following recursions hold:

$$S_n = nq^n + \sum_{k=1}^{n} \binom{n}{k} p^k q^{n-k} S_k,$$
$$p_n = \sum_{k=1}^{n} \binom{n}{k} p^k q^{n-k} p_k,$$
$$L_n = 1 - q^n + \sum_{k=1}^{n} \binom{n}{k} p^k q^{n-k} L_k.$$

Referring to the general methodology used in the analysis of algorithms (translating in terms of generating functions, then using Mellin transforms to estimate harmonic sums, see [1] for example) leads to the following result.

THEOREM 1. *Asymptotically,*

$$S_n = \frac{q}{p\log(1/p)} + P_1\left(\frac{\log n}{\log(1/p)}\right) + O(1/n),$$
$$p_n = \frac{q}{\log(1/p)} + P_2\left(\frac{\log n}{\log(1/p)}\right) + O(1/n),$$
$$L_n = \frac{\log n}{\log(1/p)} + \frac{\gamma}{\log(1/p)} - \frac{1}{2} + P_3\left(\frac{\log n}{\log(1/p)}\right) + O(1/n).$$

The $P_i(x)$'s are 1-periodic functions with amplitude of order $\exp(-\pi^2/\log(1/p))$, and γ is the Euler constant.

For $p = 1/2$, the $P_i(x)$'s have amplitude less than 10^{-5}, and $S_n \approx 1.44$, $p_n \approx 0.72$, $L_n \approx \log_2 n + 0.33$. These approximate values are quite good for $n \geq 20$. As expected, they show that the elimination phase selects a small number of survivors from a large number of contenders.

3.2. Analytic evaluation of the entire contention phase. This time, S_n, L_n and p_n denote the same quantities as before but taken at the end of the entire contention phase. With $1-r$ the geometric rate in the yield part, the entire contention phase leads to the following recursions:

$$S_n = \frac{nr}{1-(1-r)^n} q^n + \sum_{k=1}^{n} \binom{n}{k} p^k q^{n-k} S_k,$$

$$p_n = \frac{nr(1-r)^{n-1}}{1-(1-r)^n} q^n + \sum_{k=1}^{n} \binom{n}{k} p^k q^{n-k} p_k,$$

$$L_n = 1 + \frac{(1-r)^n}{1-(1-r)^n} q^n + \sum_{k=1}^{n} \binom{n}{k} p^k q^{n-k} L_k.$$

Notice that the quantities $\frac{nr}{1-(1-r)^n}$, $\frac{nr(1-r)^{n-1}}{1-(1-r)^n}$ and $\frac{(1-r)^n}{1-(1-r)^n}$ are the average number of allowed transmissions, the probability of having only one transmission and the average number of slots before first transmission in a yield contention involving n contenders, respectively.

THEOREM 2. *Asymptotically,*

$$S_n = \frac{q}{\log(1/p)} \sum_{k\geq 0} \frac{r(1-r)^k}{1-(1-r)^k q} + P_1\left(\frac{\log n}{\log(1/p)}\right) + O(1/n),$$

$$p_n = \frac{q}{\log(1/p)} \sum_{k\geq 0} \frac{r(1-r)^k}{1-(1-r)^{k+1} q} + P_2\left(\frac{\log n}{\log(1/p)}\right) + O(1/n),$$

$$L_n = \frac{\log n}{\log(1/p)} + \frac{\gamma}{\log(1/p)} - \frac{1}{2} - \frac{\log\left(\prod_{k\geq 0}(1-(1-r)^k q)\right)}{\log(1/p)} + P_3\left(\frac{\log n}{\log(1/p)}\right) + O(1/n).$$

The $P_i(x)$*'s are* 1*-periodic functions with amplitude of order* $\exp(-\pi^2/\log(1/p))$.

For the value $p = 1/2$ and $r = 1/8$, we obtain the leading terms in S_n, p_n and L_n equal to $1.0302\ldots$, $0.9713\ldots$, and $\log_2 n + 7.1393\ldots$ respectively.

3.3. Network performance analysis. As expected, there is a high probability that the contention phase selects only one survivor. The size of the overhead is $\log_2 n + O(1)$ where n is the number of contenders with the highest priority level. The throughput Thr_n is defined by

$$\mathrm{Thr}_n = \frac{p_n \mathcal{L}}{L_n + \mathcal{L} + 1},$$

where $\mathcal{L}$ is the average packet size. For n not too large (say $n \leq 32$) and for a typical value of $\mathcal{L} = 40$, the throughput is relatively stable at a value close to 0.8. This outlines the important benefit obtained from active signalling access schemes over pure carrier sense schemes (CSMA), as used by Ethernet: the throughput in CSMA rapidly collapses to 0.

Bibliography

[1] Flajolet (P.) and Sedgewick (R.). – Digital search trees revisited. *SIAM Journal on Computing*, vol. 15, n° 3, August 1986, pp. 748–767.