

Corrigenda to
“Mellin Transforms and Asymptotics: Harmonic Sums”,
 by P. Flajolet, X. Gourdon, and P. Dumas,
Theoretical Computer Science **144** (1995), pp. 3–58.

P. 11, Figure 1; P. 12, first display. [ca 2000, due to Julien Clément.]

The Mellin transform of $f(1/x)$ is $f^*(-s)$ [this corrects the third entry of Fig. 1, P. 11],

$$\frac{\frac{f(x)}{f(1/x)}}{\frac{f^*(s)}{f^*(-s)}}$$

Also [this corrects the first display on P. 12],

$$\mathcal{M}\left[f\left(\frac{1}{x}\right); s\right] = f^*(-s).$$

(The sign in the original is wrong.)

P. 20, statement of Theorem 4 and Proof. [2004-12-16, due to Manavendra Nath Mahato] Replace the three occurrences of $(\log x)^k$ by $(\log x)^{k-1}$. (Figure 4 stands as it is.) Globally, the right residue calculation, in accordance with the rest of the paper, is

$$\operatorname{Res}\left(\frac{x^{-s}}{(s-\xi)^k}\right) = \frac{(-1)^{k-1}}{(k-1)!} x^{-\xi} (\log x)^{k-1}.$$

P. 48, Example 19. [2004-10-17, due to Brigitte Vallée]

Define

$$\rho(s) = \sum_{k=1}^{\infty} \frac{r(k)}{k^s} = \sum_{m,n \geq 1} \frac{1}{(m^2 + n^2)^s}.$$

Let $\Theta(x) = \sum_{m=1}^{\infty} e^{-m^2 x^2}$. The Mellin transform of $\Theta(x^{1/2})$ is $\zeta(2s)\Gamma(s)$, and accordingly [this corrects Eq. (61) of the original; the last display on P. 29 on which this is based is correct]

$$\Theta(x^{1/2}) = \frac{\sqrt{\pi}}{2} \frac{1}{\sqrt{x}} - \frac{1}{2} + R(x),$$

where $R(x)$ is exponentially small. By squaring,

$$\Theta(x^{1/2})^2 = \frac{\pi}{4x} - \frac{\sqrt{\pi}}{2} \frac{1}{\sqrt{x}} + \frac{1}{4} + R_2(x),$$

with again $R_2(x)$ exponentially small. On the other hand, the Mellin transform of $\Theta(x^{1/2})^2$ is [this corrects the display before Eq. (61)]

$$\mathcal{M}(\Theta(x^{1/2})^2, s) = \rho(s)\Gamma(s).$$

(Equivalently, the transform of $\Theta(x)^2$ is $\frac{1}{2}\rho(s/2)\Gamma(s/2)$.) Comparing the singular expansion of $\mathcal{M}(\Theta(x^{1/2})^2, s)$ induced by the asymptotic form of $\Theta(x^{1/2})^2$ as $x \rightarrow 0$ to the exact form $\rho(s)\Gamma(s)$ of the Mellin transform shows that $\rho(s)$ is meromorphic

in the whole of \mathbb{C} , with simple poles at $s = 1, \frac{1}{2}$ *only* and singular expansion [**this corrects the last display of page 48**]

$$\rho(s) \asymp \left[\frac{\pi}{4(s-1)} \right]_{s=1} + \left[-\frac{1}{2} \frac{1}{s-\frac{1}{2}} \right]_{s=1/2} + \left[\frac{1}{4} \right]_{s=0} + [0]_{s=-1} + [0]_{s=-2} + \dots$$

(Due to a confusion in notations, the expansion computed in the paper was relative to $\rho(2s)$ rather than to $\rho(s)$; furthermore, a pole has been erroneously introduced at $s = 0$ in the last display of P. 48 In fact $\rho(0) = \frac{1}{4}$, as stated above.)