SN Sine

SN.1 Introduction

Let x be a complex variable of $\mathbb{C} \setminus \{\infty\}$.The function Sine (noted sin) is defined by the following second order differential equation

$$y(x) + \frac{\partial^2 y(x)}{\partial x^2} = 0. \tag{SN.1.1}$$

The initial conditions of SN.1.1 are given at 0 by

$$\begin{gathered} \sin(0) = 0, \\ \frac{\partial \sin(x)}{\partial x}(0) = 1. \end{gathered} \tag{SN.1.2}$$

Related function: Cosine

SN.2 Series and asymptotic expansions

SN.2.1 Asymptotic expansion at ∞.

SN.2.1.1 Exact form.

$$\sin(x) = -\frac{i}{2}\,\mathrm{e}^{\left(-\operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)x\right)} + \frac{i}{2}\,\mathrm{e}^{\left(-\operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)x\right)}.$$

SN.2.2 Taylor expansion at 0.

SN.2.2.1 First terms.

$$\begin{gathered} \sin(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \frac{1}{362880}x^9 - \frac{1}{39916800}x^{11} + \\ \frac{1}{6227020800}x^{13} - \frac{1}{1307674368000}x^{15} + \mathrm{O}\left(x^{16}\right). \end{gathered} \tag{SN.2.2.1.1}$$

SN.2.2.2 General form.

$$\sin(x) = \sum_{n=0}^{\infty} u(n) x^n. \tag{SN.2.2.2.1}$$

The coefficients $u(n)$ satisfy the recurrence

$$u(n) + \left(n^2 + 3n + 2\right) u(n+2) = 0. \tag{SN.2.2.2.2}$$

Initial conditions of SN.2.2.2.2 are given by

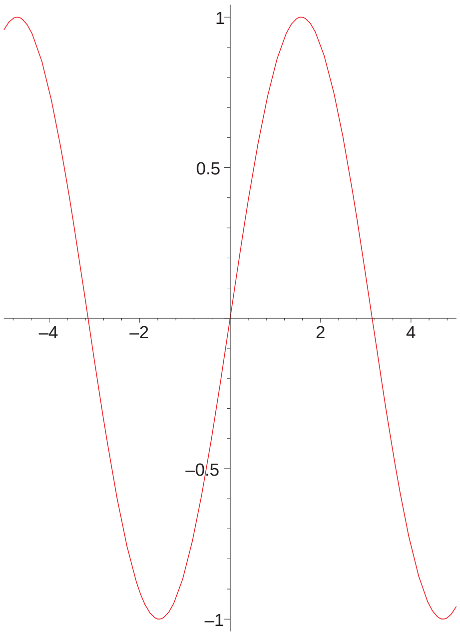
$$\begin{gathered} u(0) = 0, \\ u(1) = 1. \end{gathered} \tag{SN.2.2.2.3}$$

The recurrence SN.2.2.2.2 has the closed form solution

$$u(2n+1) = \frac{(-1)^n}{\Gamma(2n+2)}, \quad u(2n) = 0. \tag{SN.2.2.2.4}$$

SN.3 Graphs

SN.3.1 Real axis.



SN.3.2 Complex plane.