HASN Inverse Hyperbolic Sine

HASN.1 Introduction

Let x be a complex variable of $\mathbb{C} \setminus \{\infty\}$.The function Inverse Hyperbolic Sine (noted arcsinh) is defined by the following second order differential equation

$$x\frac{\partial y(x)}{\partial x} + \left(1 + x^2\right)\frac{\partial^2 y(x)}{\partial x^2} = 0. \tag{HASN.1.1}$$

The initial conditions of HASN.1.1 are given at 0 by

$$\begin{gathered} \operatorname{arcsinh}(0) = 0, \\ \frac{\partial \operatorname{arcsinh}(x)}{\partial x}(0) = 1. \end{gathered} \tag{HASN.1.2}$$

HASN.2 Series and asymptotic expansions

HASN.2.1 Asymptotic expansion at $-i$.

HASN.2.1.1 First terms.

$$\operatorname{arcsinh}(x) \approx \left(-\frac{i}{2}\pi \dots\right) + \sqrt{x - \operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)}\left(1 + i - \frac{(1+i)\left(x - \operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)\right)}{12 \operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)} + \frac{(3+3i)\left(x - \operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)\right)^2}{160 \operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)^2} - \frac{(5+5i)\left(x - \operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)\right)^3}{896 \operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)^3} + \frac{(35+35i)\left(x - \operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)\right)^4}{18432 \operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)^4} - \frac{(63+63i)\left(x - \operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)\right)^5}{90112 \operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)^5} + \frac{(231+231i)\left(x - \operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)\right)^6}{851968 \operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)^6} - \frac{(143+143i)\left(x - \operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)\right)^7}{1310720 \operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)^7} + \frac{(6435+6435i)\left(x - \operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)\right)^8}{142606336 \operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)^8} \dots\right). \tag{HASN.2.1.1.1}$$

HASN.2.1.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

HASN.2.2 Asymptotic expansion at ∞.

HASN.2.2.1 First terms.

$$\operatorname{arcsinh}(x) \approx \left(\ln(2) - \frac{1}{4x^2} + \frac{3}{32x^4} - \frac{5}{96x^6} + \frac{35}{1024x^8} + \ln\left(\frac{1}{x}\right) \dots\right). \tag{HASN.2.2.1.1}$$

HASN.2.2.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

HASN.2.3 Taylor expansion at 0.

HASN.2.3.1 First terms.

$$\operatorname{arcsinh}(x) = x - \frac{1}{6}x^3 + \frac{3}{40}x^5 - \frac{5}{112}x^7 + \frac{35}{1152}x^9 - \frac{63}{2816}x^{11} + \frac{231}{13312} x^{13} - \frac{143}{10240}x^{15} + \mathrm{O}\left(x^{16}\right). \tag{HASN.2.3.1.1}$$

HASN.2.3.2 General form.

$$\operatorname{arcsinh}(x) = \sum_{n=0}^{\infty} u(n) x^n. \tag{HASN.2.3.2.1}$$

The coefficients $u(n)$ satisfy the recurrence

$$n^2 u(n) + \left(n^2 + 3n + 2\right) u(n+2) = 0. \tag{HASN.2.3.2.2}$$

Initial conditions of HASN.2.3.2.2 are given by

$$\begin{aligned} u(0) &= 0, \\ u(1) &= 1. \end{aligned} \tag{HASN.2.3.2.3}$$

HASN.2.4 Asymptotic expansion at i.

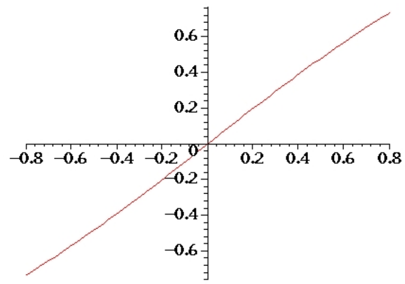
HASN.2.4.1 First terms.

$$\begin{aligned} \operatorname{arcsinh}(x) \approx \left(\frac{i}{2}\pi \dots\right) + \sqrt{x - \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)} \Bigg(1 - i - \\ \frac{(1-i)\left(x - \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)}{12 \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)} + \\ \frac{(3-3i)\left(x - \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^2}{160 \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)^2} - \\ \frac{(5-5i)\left(x - \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^3}{896 \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)^3} + \\ \frac{(35-35i)\left(x - \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^4}{18432 \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)^4} - \\ \frac{(63-63i)\left(x - \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^5}{90112 \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)^5} + \\ \frac{(231-231i)\left(x - \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^6}{851968 \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)^6} - \\ \frac{(143-143i)\left(x - \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^7}{1310720 \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)^7} + \\ \frac{(6435-6435i)\left(x - \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^8}{142606336 \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)^8} \dots \Bigg). \end{aligned} \tag{HASN.2.4.1.1}$$

HASN.2.4.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

HASN.3 Graphs

HASN.3.1 Real axis.



HASN.3.2 Complex plane.