CS Cosine

CS.1 Introduction

Let x be a complex variable of $\mathbb{C} \setminus \{\infty\}$.The function Cosine (noted cos) is defined by the following second order differential equation

$$y(x) + \frac{\partial^2 y(x)}{\partial x^2} = 0. \tag{CS.1.1}$$

The initial conditions of CS.1.1 are given at 0 by

$$\begin{aligned} \cos(0) &= 1, \\ \frac{\partial \cos(x)}{\partial x}(0) &= 0. \end{aligned} \tag{CS.1.2}$$

Related function: Sine

CS.2 Series and asymptotic expansions

CS.2.1 Asymptotic expansion at ∞.

CS.2.1.1 Exact form.

$$\cos(x) = \frac{e^{(-\operatorname{RootOf}_{\xi,2}(1+\xi^2)x)}}{2} + \frac{e^{(-\operatorname{RootOf}_{\xi,1}(1+\xi^2)x)}}{2}.$$

CS.2.2 Taylor expansion at 0.

CS.2.2.1 First terms.

$$\begin{aligned} \cos(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \frac{1}{40320}x^8 - \frac{1}{3628800}x^{10} + \\ \frac{1}{479001600}x^{12} - \frac{1}{87178291200}x^{14} + \mathrm{O}(x^{16}). \end{aligned} \tag{CS.2.2.1.1}$$

CS.2.2.2 General form.

$$\cos(x) = \sum_{n=0}^{\infty} u(n)x^n. \tag{CS.2.2.2.1}$$

The coefficients $u(n)$ satisfy the recurrence

$$u(n) + (n^2 + 3n + 2)u(n+2) = 0. \tag{CS.2.2.2.2}$$

Initial conditions of CS.2.2.2.2 are given by

$$\begin{aligned} u(1) &= 0, \\ u(0) &= 1. \end{aligned} \tag{CS.2.2.2.3}$$

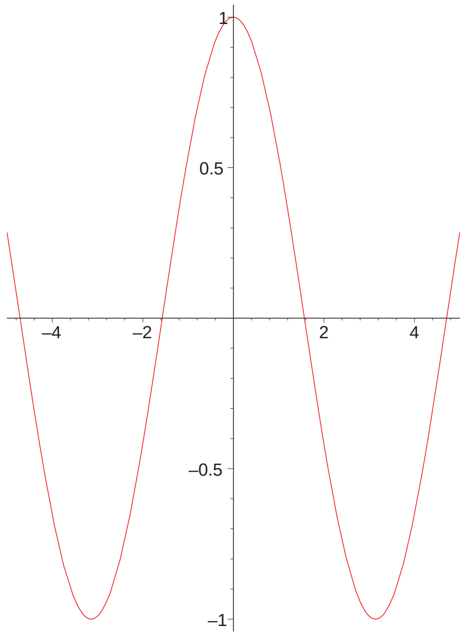
The recurrence CS.2.2.2.2 has the closed form solution

$$u(2n+1) = 0,$$

(CS.2.2.2.4)
$$u(2n) = \frac{(-1)^n}{\Gamma(2n+1)}.$$

CS.3 Graphs

CS.3.1 Real axis.



CS.3.2 Complex plane.