ACCS Inverse Cosecant

ACCS.1 Introduction

Let x be a complex variable of $\mathbb{C} \setminus \{0\}$.The function Inverse Cosecant (noted arccsc) is defined by the following second order differential equation

$$\left(2x^2-1\right)\frac{\partial y(x)}{\partial x}+\left(x^3-x\right)\frac{\partial^2 y(x)}{\partial x^2}=0. \tag{ACCS.1.1}$$

The initial conditions of ACCS.1.1 at 0 are not simple to state, since 0 is a (regular) singular point.

Related functions: Inverse Hyperbolic Secant,Inverse Secant

ACCS.2 Series and asymptotic expansions

ACCS.2.1 Asymptotic expansion at -1.

ACCS.2.1.1 First terms.

$$\begin{aligned}\operatorname{arccsc}(x) \approx \left(\frac{-\pi}{2}\dots\right)+\sqrt{x+1}\Bigg(&i\sqrt{2}+\frac{5i}{12}(x+1)\sqrt{2}+\\ &\frac{43i}{160}(x+1)^2\sqrt{2}+\frac{177i}{896}(x+1)^3\sqrt{2}+\frac{2867i}{18432}(x+1)^4\sqrt{2}+\\ &\frac{11531i}{90112}(x+1)^5\sqrt{2}+\frac{92479i}{851968}(x+1)^6\sqrt{2}+\\ &\frac{74069i}{786432}(x+1)^7\sqrt{2}+\frac{11857475i}{142606336}(x+1)^8\sqrt{2}\dots\Bigg).\end{aligned} \tag{ACCS.2.1.1.1}$$

ACCS.2.1.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

ACCS.2.2 Asymptotic expansion at 0.

ACCS.2.2.1 First terms.

$$\begin{aligned}\operatorname{arccsc}(x) \approx \Bigg(&\\ &-i\ln(2)+\frac{\pi}{2}-\frac{i}{4}x^2-\frac{3i}{32}x^4-\frac{5i}{96}x^6-\frac{35i}{1024}x^8-i\ln(x)\dots\Bigg).\end{aligned} \tag{ACCS.2.2.1.1}$$

ACCS.2.2.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

ACCS.2.3 Asymptotic expansion at 1.

ACCS.2.3.1 First terms.

$$\operatorname{arccsc}(x) \approx \left(\frac{\pi}{2}\dots\right) + \sqrt{x-1}\left(-\sqrt{2} + \frac{5(x-1)\sqrt{2}}{12} - \frac{43(x-1)^2\sqrt{2}}{160} + \frac{177(x-1)^3\sqrt{2}}{896} - \frac{2867(x-1)^4\sqrt{2}}{18432} + \frac{11531(x-1)^5\sqrt{2}}{90112} - \frac{92479(x-1)^6\sqrt{2}}{851968} + \frac{74069(x-1)^7\sqrt{2}}{786432} - \frac{11857475(x-1)^8\sqrt{2}}{142606336}\dots\right). \tag{ACCS.2.3.1.1}$$

ACCS.2.3.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).