

Planar Maps and Composition Schemes

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Abstract

This talk is concerned with presenting the enumerative theory of planar maps, following Tutte's original approach. Combinatorial proofs of many beautiful formulæ discovered by Tutte have been given recently (cf. last year's talk). However, in "less beautiful" cases, one is invariably back to decompositions and generating series. In other words, the best tools still are those introduced by Tutte and Brown in the 1960's. The decompositions by "deletion/contraction" of edges translate into quadratic bivariate discrete differential equations, that transform into algebraic equations by the (rather miraculous) "quadratic method." Decompositions by "composition" of maps translate into composition schemes. From the viewpoint of singularity analysis, these schemes are all of the same type: the critical composition of two singularities in $(\rho - x)^{3/2}$. The goal of this presentation is especially to show where the composition schemes and the random generation algorithm reported on in Cyril Banderier's companion talk stem from.