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On Random Graph Homomorphisms into $\mathbb{Z}$

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Abstract

The study of Lipschitz functions on graphs and metric spaces is rather advanced. Uniform measure on graph homomorphisms into $\mathbb{Z}$ provides a model for looking at typical Lipschitz functions. Given a bipartite connected finite graph $G = (V, E)$ and a vertex $v_0 \in V$, we consider a uniform probability measure on the set of graph homomorphisms $f : V \to \mathbb{Z}$ satisfying $f(v_0) = 0$. This measure can be viewed as a G-indexed random walk on $\mathbb{Z}$, generalizing both the usual time-indexed random walk and tree-indexed random walk. We will present several general inequalities for G-indexed random walks, including an upper bound on fluctuations implying that the distance $d\big(f(u), f(v)\big)$ between $f(u)$ and $f(v)$, is stochastically dominated by the distance to 0 of a simple random walk on $\mathbb{Z}$ having run for $d(u, v)$ steps. We will also discuss various special cases, some conjectures and algorithmic aspects of these models.