# A singular mathematician and the singularity analysis of generating functions: In memory of Philippe Flajolet and in tribute to his work

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# Analytic combinatorics — or not:

Combinatorialists use recurrence, generating functions, and such transformations as the Vandermonde convolution; others, to my horror, use contour integrals, differential equations, and other resources of mathematical analysis.

J. Riordan

# Analytic combinatorics (as embraced by Philippe Flajolet):

encode objects under study as generating functions, and use analytic properties of those functions to obtain asymptotic information of interest

- location of singularities determines rough growth rate of coefficients
- nature of singularities determines fine-scale behavior

# Singularity analysis and analytic combinatorics:

- most of analytic combinatorics relies on singularity analysis
  - see *Analytic Combinatorics* of Philippe Flajolet and Bob Sedgewick (the most frequently cited work of PF, according to Google Scholar)
- most of Philippe Flajolet's work relies directly or indirectly on singularity analysis

# Philippe Flajolet and singularity analysis:

- many papers, even among those classified in singularity analysis
- will concentrate on just one: P. Flajolet and AO, "Singularity analysis of generating functions," *SIAM J. Disc. Math.*, vol. 3, no. 2, May 1990, pp. 216–240
- most widely cited paper of Philippe Flajolet (and AO), according to Google Scholar

# Transfer theorems:

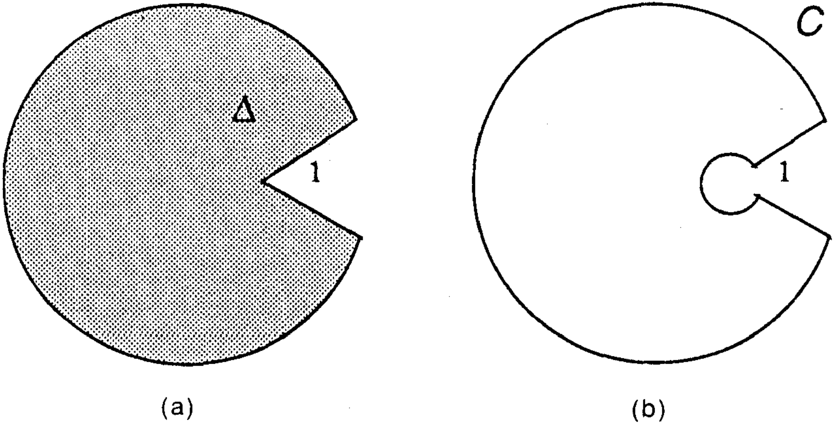
plane. For instance, under suitable analytic conditions, an expansion

$$f(z) \sim \frac{1}{\sqrt{1-z}}\left(\frac{c_0}{\sqrt{\log(1-z)^{-1}}} + \frac{c_1}{\log(1-z)^{-1}} + \frac{c_2}{\sqrt{\log^3(1-z)^{-1}}} + \cdots\right) \qquad (z \to 1) \tag{1.2a}$$

"transfers" to coefficients as

$$f_n \sim \frac{1}{\sqrt{\pi n}}\left(\frac{c_0}{\sqrt{\log n}} + \frac{c_1}{\log n} + \frac{c_2'}{\sqrt{\log^3 n}} + \cdots\right) \qquad (n \to \infty) \tag{1.2b}$$

# Important tool:



# Importance of singularity analysis paper:

- provides a tool that is easy to use, even by non-experts
- general and powerful
- complements other general methods:
  - Darboux method
  - Tauberian theorems
  - Hayman admissibility

# Continuing progress in singularity analysis:

- many ongoing advances
- perhaps most noteworthy: multivariate asymptotics work of Baryshnikov, Pemantle, and Wilson
- surely much more to come!
- extends and reinforces Philippe Flajolet's manifold contributions