Early works of Philippe Flajolet on protocols and telecommunication

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Recalling my (old) collaboration with Philippe

- 1983: internship in INRIA in PF Algorithm group
- 1985: Publication of my first IEEE IT paper
    - Fayolle, PF, Hofri, PJ, The evaluation of packet transmission characteristics in a multi-access channel with stack collision resolution protocol » 1985
- 1989: PhD, PF as director
    - 2nd PF's PhD student, 1st one was Mireille Régnier (*)
- 1989-1998: Working on tree and protocols AofA in PF group
    - Parallel extensive collaboration with Wojtek:
        - dePoissonization, digital search trees, etc
- 1998: Foundation of Hipercom group dedicated to high performance telecommunication algorithms.

(*) In INRIA, Algo group, after 1981

Flajolet work applied to telecommunication

- **Approximate counting** is now a strong tool applicable to
  - internet router flow monitoring
  - Cyber-attack detection
  - But originally not designed for telecommunication and protocols

- Here we talk about PF work originally designed for **telecommunication**
  - Collision Resolution Algorithms

The collision resolution problem

- Assume a time slotted channel
  - Multiple access:
    - All users connected
    - All users listen slot feedback
  - At every slot:
    - If no contender: empty slot
    - If two or more contenders: collision slot (data are lost)
    - If one contender: successful transmission

ALOHA

- On each slot
  - Each Active User (with pending packets to send) contends with probability p
  - If *n* active users: $P(\text{collision}) = 1 - O(n(1-p)^n) \to 1$
  - Thus $P(\text{success}) \to 0$
    - The throughput tends to 0 when *n* tends to infinity
- Consequence:
  - ALOHA deadlocks when *n* tends to infinity

Abramson, « the ALOHA system, another alternative to computer communications », 1968

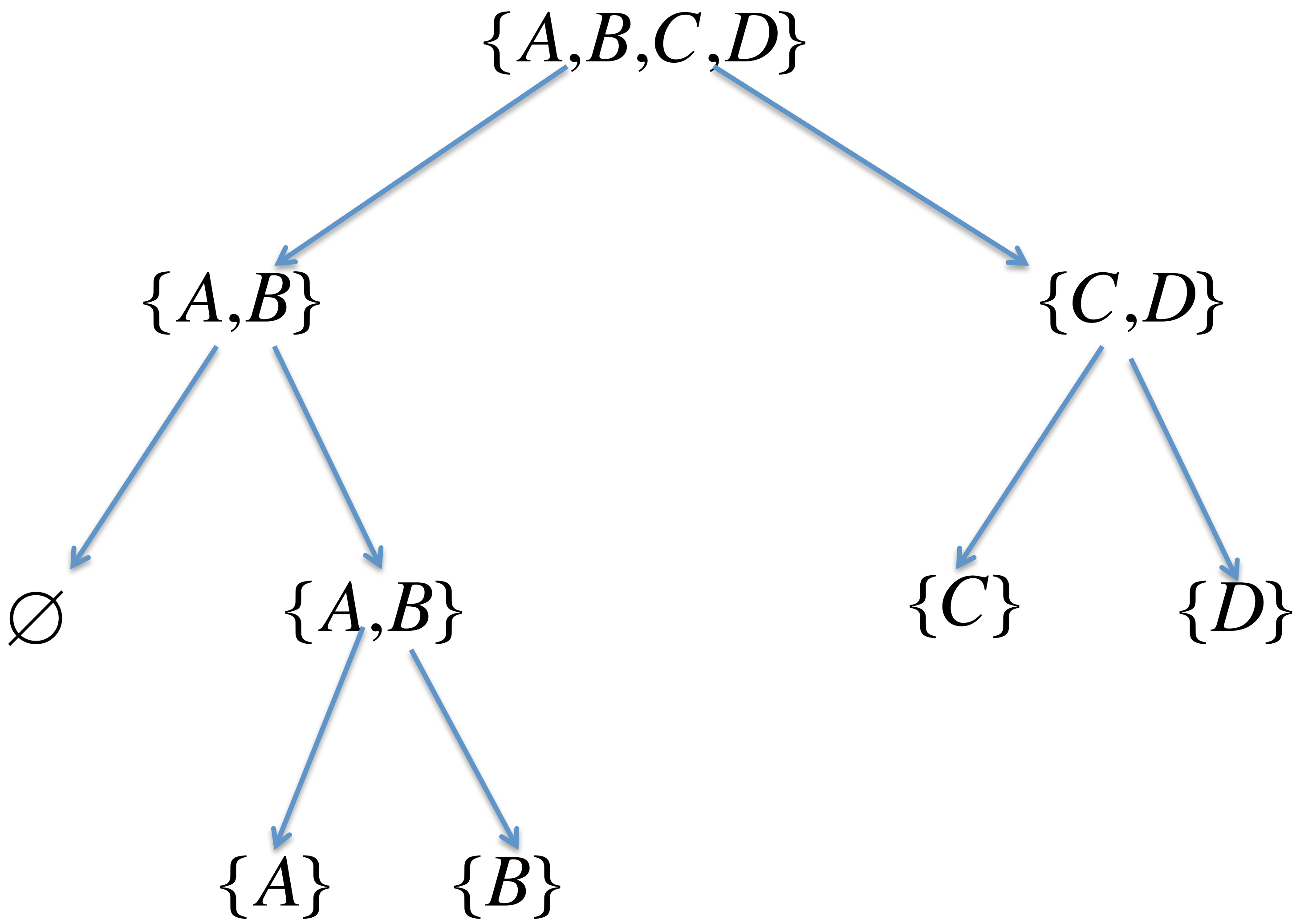
How to remove ALOHA deadlock

- Throughput $P(\text{success}) = np(1-p)^{n-1}$
  - Optimal when $p = \frac{1}{n}$
  - In this case $P(\text{success}) \rightarrow e^{-1}$

- Kind of Approximate counting via *leader election* on a collision channel
  - PF, Greenberg, Ladner « Estimating the multiplicities of conflicts to speed their resolution in multiple access channels», 1987 [77 citations]

The tree algorithm

- Invented by Capetanakis in 1978
  - Collision resolution via random spliting
  - After each collision a binary tree is created
    - Contenders toss coins
      - Heads contend on first subtree
      - Tails contend on second subtree



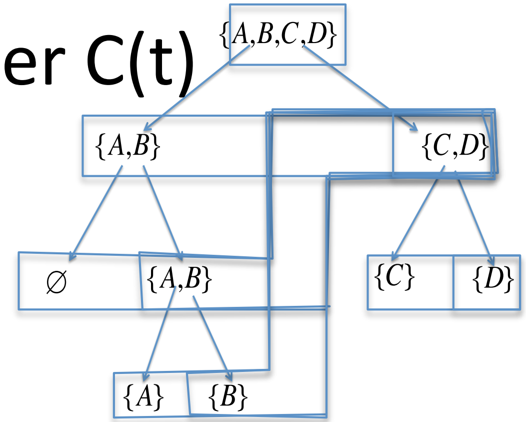
A collision resolution tree
Tree reads left depth first

Stack algorithm vs tree algorithm

Tsybakov-Vvedenskaya 1980

Each active user manage a counter C(t)

- -initialized at zero when packet arrives.
- -user transmits when C(t)=0
- -if collision user sets C(t+1)=toss (0 or 1)
- -when waiting for retransmission: C(t)>0
  - if collision C(t+1)=C(t)+1
  - if non collision C(t+1)=C(t)-1



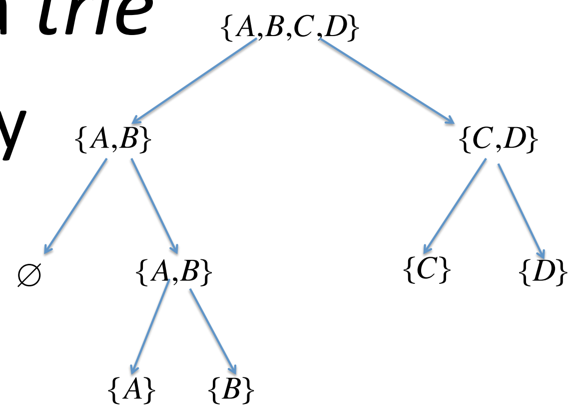
Collision resolution interval

ABCD	**AB**	**-**	**AB**	**A**	**B**	**CD**	**C**	**D**
	CD	AB	CD	B	CD		D	
		CD		CD				

Tree and stack algorithms are the same!

Tree algorithm vs *trie*

- A collision resolution tree is a *trie*
  - Toss sequence is contender key
  - Leaf capacity is one



  - Average collision resolution interval length

$$L_n = 1 + \sum_k 2^{-n} \binom{n}{k} \left(L_k + L_{n-k} \right)$$

Poisson pgf: $L(z) = 1 - 2(1+z)e^{-z} + 2L\left(\frac{z}{2}\right)$

$$L_n = \frac{n}{0.346573}(1 + O(10^{-6}))$$

Biased toss (p,q)

$$L(z) = 1 - 2(1+z)e^{-z} + L(pz) + L(qz)$$

$$L_n = \frac{2n}{-p\log p - q\log q}(1 + r(\log n))$$

Performance of tree algorithm

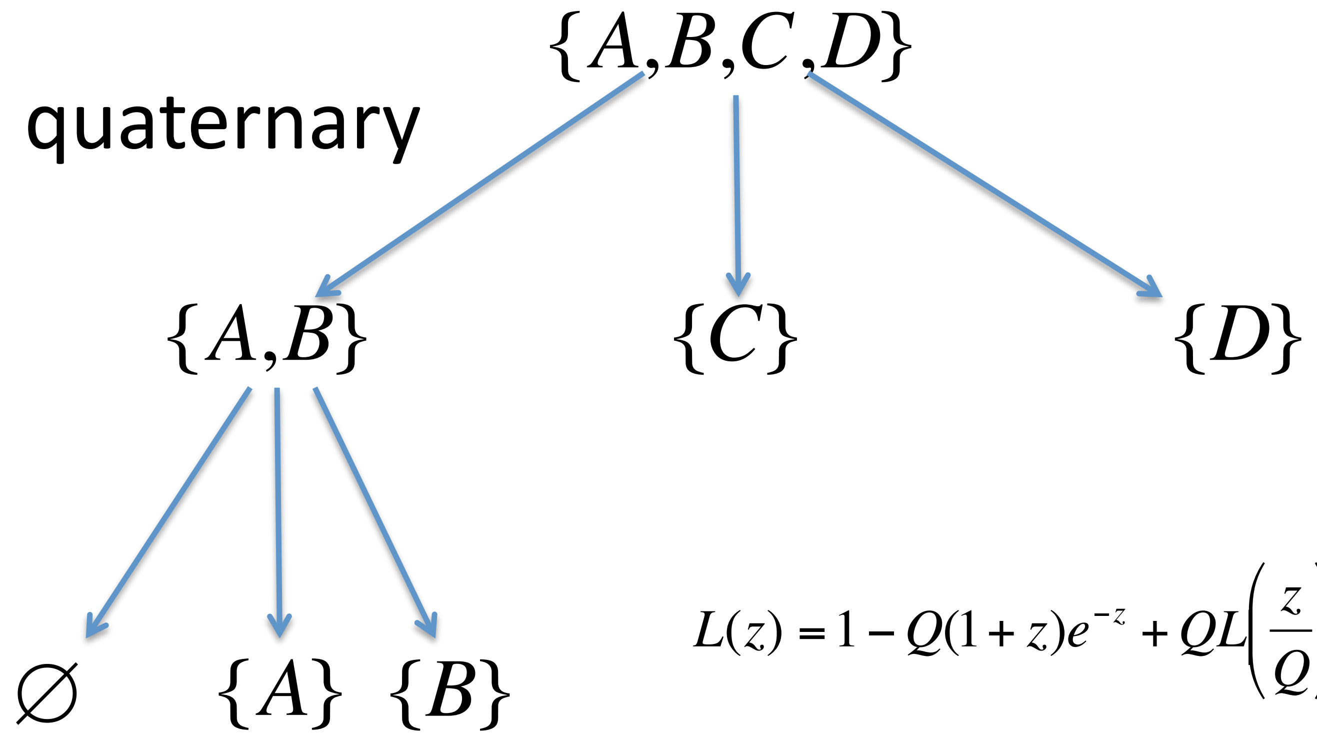
- Average success rate per interval is $\frac{n}{L_n}$
  - If new packet arrivals are blocked
- Let λ be the average per slot packet generation rate

- If $\lambda > \limsup_{n\to\infty} \frac{n}{L_n}$
  - Tree algorithm is unstable
- If $\lambda < \liminf_{n\to\infty} \frac{n}{L_n}$
  - Tree algorithm is stable $\lambda_{\max} \approx 0.34657\cdots$
  - Exact evaluation of $\lambda_{\max}$ is an open problem due to periodic terms

- Massey « collision resolution algorithms and random access communications, 1981.
- Fayolle, Hofri, « On the Capacity of a Collision Resolution Channel under Stack based Collision Resolution Algorithm » 1983

What about Q-ary tree?

- Ternary, quaternary



$$L(z) = 1 - Q(1+z)e^{-z} + QL\left(\frac{z}{Q}\right)$$

$$L_n = \frac{Qn}{\log Q}(1 + r(\log n))$$

$$\lambda_{\max} = \frac{\log Q}{Q}(1 + O(10^{-6}))$$

Optimal degree Q

$$\lambda_{\max} \approx \frac{\log Q}{Q}$$

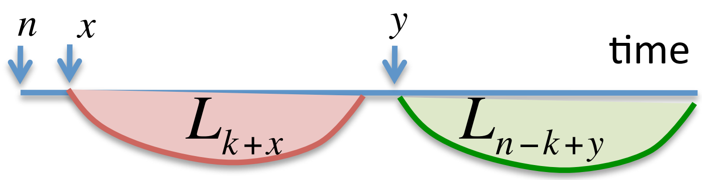
- Optimal Q would be Q=*e* for $\lambda_{\max} = e^{-1}$
- Integer optimal is Q=3 for $\lambda_{\max} \approx 0.366204 < \mathrm{e}^{-1} = 0.367879$
- Conjecture of the 80's:
  – is 1/e the optimal throughput?
  – Answer in PF work at the end of the talk.

Issue: an unbiased ternary toss made via binary coins…

Unblocking new packet arrivals

- New users participate to current resolution
  - If arrivals per slot are i.i.d

$$L_n = 1 + \sum_{x,y} P(x)P(y) \sum_k 2^{-n} \binom{n}{k} \left(L_{k+x} + L_{n-k+y} \right)$$



  - If arrivals are Poisson of rate λ

$$L(z) = 1 - 2L(\lambda)(1+z)e^{-z} - L'(\lambda)ze^{-z} + 2L\left(\frac{z}{2} + \lambda\right)$$

    - With biased toss (p,q)

$$L(z) = 1 - 2L(\lambda)(1+z)e^{-z} - L'(\lambda)ze^{-z} + L(pz + \lambda) + L(qz + \lambda)$$

    - Fayolle, PF, Hofri, « On a functional equation arising in the analysis of a protocol for a multi-access broadcast channel, 1982 » [69 citations]

Unblocking new packet arrivals

- Solving $$L(z) = 1 - L(\lambda) f(z) - L'(\lambda) g(z) + 2L \circ h(z)$$
  - With $$h(z) = \frac{z}{2} + \lambda$$
  - Iterative scheme $$L(z) = 1 - L(\lambda)\mathbf{H}f(z) - L'(\lambda)\mathbf{H}g(z)$$

$$\begin{cases} \mathbf{H}f = \sum_{k\geq 0} 2^k f \circ h^k \\ \mathbf{H}g = \sum_{k\geq 0} 2^k g \circ h^k \end{cases}$$

Diverging, but we cope with this.

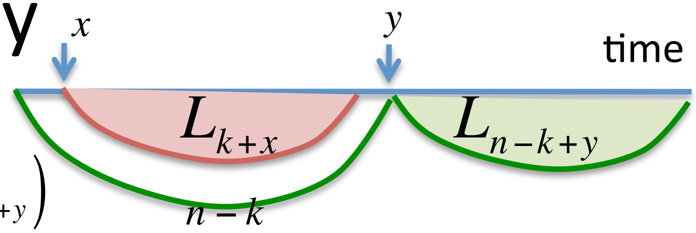
  - General solution

$$\begin{bmatrix} L(\lambda) \\ L'(\lambda) \end{bmatrix} = \begin{bmatrix} 1 - \mathbf{H}f(\lambda) & -\mathbf{H}g(\lambda) \\ -(\mathbf{H}f)'(\lambda) & 1 - (\mathbf{H}g)'(\lambda) \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

  - Matrix is degenerated for $\lambda_{\max} = 0.360177\cdots < e^{-1}$

Average packet delay

- Telecom people want packet delay analysis
- average cumulated packet delay



$$W_n = n + \sum_{x,y} P(x)P(y) \sum_k 2^{-n} \binom{n}{k} \left(W_{k+x} + (n-k)L_{k+x} + W_{n-k+y} \right)$$

$$W(z) = z + \frac{z}{2} L(\frac{z}{2} + \lambda) - L(\lambda)\frac{z}{2} e^{-z} - 2W(\lambda)(1+z)e^{-z} - W'(\lambda) z e^{-z} + 2W\left(\frac{z}{2} + \lambda\right)$$

- Resolution via application of operator **H**
  - Average packet delay $\frac{W(\lambda)}{\lambda L(\lambda)}$
  - First full analysis of a collision resolution algorithm
    - Fayolle, PF, Hofri, PJ, « The evaluation of packet transmission characteristics in a multi-access channel with stack collision resolution protocol » 1985 [99 citations]

Q-ary tree algorithm with unblocked new packet arrivals

$$L(z) = 1 - QL(\lambda)(1+z)e^{-z} - L'(\lambda)ze^{-z} + QL\left(\frac{z}{Q} + \lambda\right)$$

- Resolution is similar as for Q=2
- Optimal is Q=3 with $\lambda_{\max} = 0.401599\cdots > e^{-1}$
  - Therefore 1/e is not the ultimate throughput
    - Flajolet Mathys, « Q-ary collision resolution algorithms in random-access systems with free or blocked channel access » 1985 [183 citations]