A Happy New Year 2010

Consider the **integer sequence** (p_n), which starts as

$$2, 144, 96768, 268240896, 2111592333312, 37975288540299264, \ldots$$

and is defined by sums over the **square lattice**,

$$p_n := (-1)^{n+1}(4n+3)! \left[\int_0^1 \frac{dt}{\sqrt{1-t^4}}\right]^{-4n-4} \sum_{a,b=-\infty}^{+\infty} \left[(2a+1)+(2b+1)\sqrt{-1}\right]^{-4n-4}.$$

The following **continued fraction** expansion holds:

$$\sum_{n=0}^{\infty} p_n z^n = \cfrac{2}{1-2\cdot 2^2(2^2+5)\,z - \cfrac{2\cdot 3^2\cdot 4^2\cdot 5^2\cdot 6\,z^2}{1-2\cdot 6^2(6^2+5)\,z - \cfrac{6\cdot 7^2\cdot 8^2\cdot 9^2\cdot 10\,z^2}{1-2\cdot 10^2(10^2+5)\,z - \ddots}}}.$$

[A follow up to R. Bacher and P. Flajolet, *The Ramanujan Journal*, 2010, in press.]

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