SN Sine

SN.1 Introduction

Let x be a complex variable of $\mathbb{C}\setminus\{\infty\}$.The function Sine (noted sin) is defined by the following second order differential equation

$$y(x)+\frac{\partial^2 y(x)}{\partial x^2}=0. \tag{SN.1.1}$$

The initial conditions of SN.1.1 are given at 0 by

$$\begin{aligned}\sin(0)&=0,\\ \frac{\partial\sin(x)}{\partial x}(0)&=1.\end{aligned} \tag{SN.1.2}$$

Related function: Cosine

SN.2 Series and asymptotic expansions

SN.2.1 Asymptotic expansion at ∞.

SN.2.1.1 Exact form.

$$\sin(x)=-\frac{i}{2}\,\mathrm{e}^{\left(-\operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)x\right)}+\frac{i}{2}\,\mathrm{e}^{\left(-\operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)x\right)}.$$

SN.2.2 Taylor expansion at 0.

SN.2.2.1 First terms.

$$\begin{aligned}\sin(x)=x-\frac{1}{6}x^3+\frac{1}{120}x^5-\frac{1}{5040}x^7+\frac{1}{362880}x^9-\frac{1}{39916800}x^{11}+\\ \frac{1}{6227020800}x^{13}-\frac{1}{1307674368000}x^{15}+\mathrm{O}\left(x^{16}\right).\end{aligned} \tag{SN.2.2.1.1}$$

SN.2.2.2 General form.

$$\sin(x)=\sum_{n=0}^{\infty}u(n)x^n. \tag{SN.2.2.2.1}$$

The coefficients $u(n)$ satisfy the recurrence

$$u(n)+\left(n^2+3n+2\right)u(n+2)=0. \tag{SN.2.2.2.2}$$

Initial conditions of SN.2.2.2.2 are given by

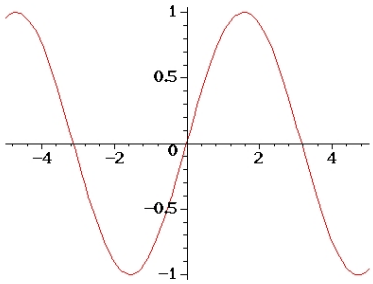
$$\begin{aligned}u(0)&=0,\\ u(1)&=1.\end{aligned} \tag{SN.2.2.2.3}$$

The recurrence SN.2.2.2.2 has the closed form solution

$$u(2n+1) = \frac{(-1)^n}{\Gamma(2n+2)}, \qquad u(2n) = 0. \tag{SN.2.2.2.4}$$

SN.3 Graphs

SN.3.1 Real axis.



SN.3.2 Complex plane.