HSN Hyperbolic Sine

HSN.1 Introduction

Let x be a complex variable of $\mathbb{C} \setminus \{\infty\}$.The function Hyperbolic Sine (noted sinh) is defined by the following second order differential equation

$$-y(x) + \frac{\partial^2 y(x)}{\partial x^2} = 0. \tag{HSN.1.1}$$

The initial conditions of HSN.1.1 are given at 0 by

$$\begin{aligned} \sinh(0) &= 0, \\ \frac{\partial \sinh(x)}{\partial x}(0) &= 1. \end{aligned} \tag{HSN.1.2}$$

Related function: Hyperbolic Cosine

HSN.2 Series and asymptotic expansions

HSN.2.1 Taylor expansion at 0.

HSN.2.1.1 First terms.

$$\begin{aligned} \sinh(x) = x + \frac{1}{6}x^3 + \frac{1}{120}x^5 + \frac{1}{5040}x^7 + \frac{1}{362880}x^9 + \frac{1}{39916800}x^{11} + \\ \frac{1}{6227020800}x^{13} + \frac{1}{1307674368000}x^{15} + \mathrm{O}(x^{16}). \end{aligned} \tag{HSN.2.1.1.1}$$

HSN.2.1.2 General form.

$$\sinh(x) = \sum_{n=0}^{\infty} u(n) x^n. \tag{HSN.2.1.2.1}$$

The coefficients $u(n)$ satisfy the recurrence

$$-u(n) + \left(n^2 + 3n + 2\right) u(n+2) = 0. \tag{HSN.2.1.2.2}$$

Initial conditions of HSN.2.1.2.2 are given by

$$\begin{aligned} u(0) &= 0, \\ u(1) &= 1. \end{aligned} \tag{HSN.2.1.2.3}$$

The recurrence HSN.2.1.2.2 has the closed form solution

$$\begin{aligned} u(2n+1) &= \frac{1}{\Gamma(2n+2)}, \\ u(2n) &= 0. \end{aligned} \tag{HSN.2.1.2.4}$$

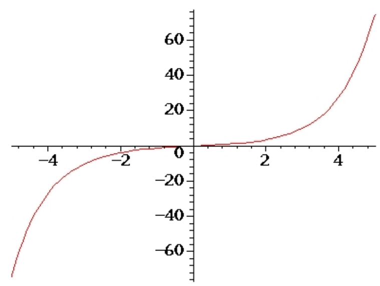
HSN.2.2 Asymptotic expansion at ∞.

HSN.2.2.1 Exact form.

$$\sinh(x) = \frac{e^x}{2} - \frac{e^{(-x)}}{2}.$$

HSN.3 Graphs

HSN.3.1 Real axis.



HSN.3.2 Complex plane.