HCS Hyperbolic Cosine

HCS.1 Introduction

Let x be a complex variable of $\mathbb{C} \setminus \{\infty\}$.The function Hyperbolic Cosine (noted cosh) is defined by the following second order differential equation

$$-y(x) + \frac{\partial^2 y(x)}{\partial x^2} = 0. \tag{HCS.1.1}$$

The initial conditions of HCS.1.1 are given at 0 by

$$\begin{aligned} \cosh(0) &= 1, \\ \frac{\partial \cosh(x)}{\partial x}(0) &= 0. \end{aligned} \tag{HCS.1.2}$$

Related function: Hyperbolic Sine

HCS.2 Series and asymptotic expansions

HCS.2.1 Asymptotic expansion at ∞.

HCS.2.1.1 Exact form.

$$\cosh(x) = \frac{e^x}{2} - \frac{e^{(-x)}}{2}.$$

HCS.2.2 Taylor expansion at 0.

HCS.2.2.1 First terms.

$$\begin{aligned} \cosh(x) = 1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \frac{1}{720}x^6 + \frac{1}{40320}x^8 + \frac{1}{3628800}x^{10} + \\ \frac{1}{479001600}x^{12} + \frac{1}{87178291200}x^{14} + \mathrm{O}\big(x^{16}\big). \end{aligned} \tag{HCS.2.2.1.1}$$

HCS.2.2.2 General form.

$$\cosh(x) = \sum_{n=0}^{\infty} u(n) x^n. \tag{HCS.2.2.2.1}$$

The coefficients $u(n)$ satisfy the recurrence

$$-u(n) + \big(n^2 + 3n + 2\big) u(n+2) = 0. \tag{HCS.2.2.2.2}$$

Initial conditions of HCS.2.2.2.2 are given by

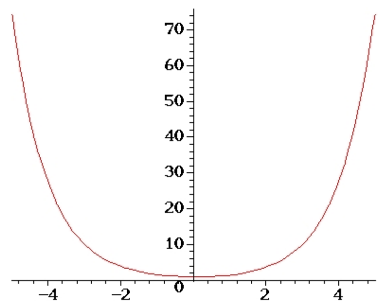
$$\begin{aligned} u(1) &= 0, \\ u(0) &= 1. \end{aligned} \tag{HCS.2.2.2.3}$$

The recurrence HCS.2.2.2.2 has the closed form solution

$$u(2n) = \frac{1}{\Gamma(2n+1)},\quad u(2n+1) = 0. \tag{HCS.2.2.2.4}$$

HCS.3 Graphs

HCS.3.1 Real axis.



HCS.3.2 Complex plane.