HASC Inverse Hyperbolic Secant

HASC.1 Introduction

Let x be a complex variable of $\mathbb{C}\setminus\{0\}$.The function Inverse Hyperbolic Secant (noted arcsech) is defined by the following second order differential equation

$$\left(2x^2-1\right)\frac{\partial y(x)}{\partial x}+\left(x^3-x\right)\frac{\partial^2 y(x)}{\partial x^2}=0. \qquad \text{(HASC.1.1)}$$

The initial conditions of HASC.1.1 at 0 are not simple to state, since 0 is a (regular) singular point.

Related functions: Inverse Secant,Inverse Cosecant

HASC.2 Series and asymptotic expansions

HASC.2.1 Asymptotic expansion at 1.

HASC.2.1.1 First terms.

$$\begin{aligned}\operatorname{arcsech}(x) \approx \sqrt{x-1}\Bigg(&\frac{47442055i}{637534208}\sqrt{2}(x-1)^9 - i\sqrt{2} - \\ &\frac{24295375159i}{429496729600}\sqrt{2}(x-1)^{12} + \frac{3109879375897i}{68169720922112}\sqrt{2}(x-1)^{15} - \\ &\frac{43i}{160}\sqrt{2}(x-1)^2 + \frac{5i}{12}\sqrt{2}(x-1) + \frac{74069i}{786432}\sqrt{2}(x-1)^7 + \\ &\frac{1518418695i}{24696061952}\sqrt{2}(x-1)^{11} - \frac{92479i}{851968}\sqrt{2}(x-1)^6 - \\ &\frac{126527543i}{1879048192}\sqrt{2}(x-1)^{10} - \frac{2867i}{18432}\sqrt{2}(x-1)^4 - \\ &\frac{11857475i}{142606336}\sqrt{2}(x-1)^8 + \frac{177i}{896}\sqrt{2}(x-1)^3 + \\ &\frac{11531i}{90112}\sqrt{2}(x-1)^5 - \frac{777467420263i}{15942918602752}\sqrt{2}(x-1)^{14} + \\ &\frac{97182800711i}{1855425871872}\sqrt{2}(x-1)^{13}\dots\Bigg).\end{aligned} \qquad \text{(HASC.2.1.1.1)}$$

HASC.2.1.2 General form.

$$\operatorname{arcsech}(x) \approx \sqrt{x-1}\sum_{n=0}^{\infty} u(n)(x-1)^n. \qquad \text{(HASC.2.1.2.1)}$$

The coefficients $u(n)$ satisfy the recurrence

$$2u(n)\left(\frac{1}{2}+n\right)n + u(n-1)\left(-\frac{1}{2}+n\right)\left(-\frac{1}{2}+3n\right) + u(n-2)\left(-\frac{3}{2}+n\right)\left(-\frac{1}{2}+n\right) = 0. \qquad \text{(HASC.2.1.2.2)}$$

Initial conditions of HASC.2.1.2.2 are given by

$$u(0) = -i\sqrt{2}, \qquad u(1) = \frac{5i}{12}\sqrt{2}. \tag{HASC.2.1.2.3}$$

HASC.2.2 Asymptotic expansion at -1.

HASC.2.2.1 First terms.

$$\operatorname{arcsech}(x) \approx (i\pi \ldots) + \sqrt{x+1}\Bigg(-\sqrt{2} - \frac{5(x+1)\sqrt{2}}{12} - \frac{43(x+1)^2\sqrt{2}}{160} - \frac{177(x+1)^3\sqrt{2}}{896} - \frac{2867(x+1)^4\sqrt{2}}{18432} - \frac{11531(x+1)^5\sqrt{2}}{90112} - \frac{92479(x+1)^6\sqrt{2}}{851968} - \frac{74069(x+1)^7\sqrt{2}}{786432} - \frac{11857475(x+1)^8\sqrt{2}}{142606336} \ldots\Bigg). \tag{HASC.2.2.1.1}$$

HASC.2.2.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

HASC.2.3 Asymptotic expansion at 0.

HASC.2.3.1 First terms.

$$\operatorname{arcsech}(x) \approx \left(\frac{x^2}{4} + \frac{3x^4}{32} + \frac{5x^6}{96} + \frac{35x^8}{1024} + \ln(x) + \ln(2) \ldots\right). \tag{HASC.2.3.1.1}$$

HASC.2.3.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).