HACT Inverse Hyperbolic Cotangent

HACT.1 Introduction

Let x be a complex variable of $\mathbb{C} \setminus \{-1, 1\}$.The function Inverse Hyperbolic Cotangent (noted arccoth) is defined by the following second order differential equation

$$2x\frac{\partial y(x)}{\partial x} + \left(x^2 - 1\right)\frac{\partial^2 y(x)}{\partial x^2} = 0. \tag{HACT.1.1}$$

The initial conditions of HACT.1.1 are given at 0 by

$$\begin{aligned} \operatorname{arccoth}(0) &= -\frac{i}{2}\pi, \\ \frac{\partial \operatorname{arccoth}(x)}{\partial x}(0) &= 1. \end{aligned} \tag{HACT.1.2}$$

Related function: Inverse Hyperbolic Tangent

HACT.2 Series and asymptotic expansions

HACT.2.1 Taylor expansion at 0.

HACT.2.1.1 First terms.

$$\begin{aligned} \operatorname{arccoth}(x) = &-\frac{i}{2}\pi + x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \frac{1}{7}x^7 + \frac{1}{9}x^9 + \frac{1}{11}x^{11} + \frac{1}{13}x^{13} + \frac{1}{15} \\ &x^{15} + \mathrm{O}\left(x^{16}\right). \end{aligned} \tag{HACT.2.1.1.1}$$

HACT.2.1.2 General form.

$$\operatorname{arccoth}(x) = \sum_{n=0}^{\infty} u(n)x^n. \tag{HACT.2.1.2.1}$$

The coefficients $u(n)$ satisfy the recurrence

$$nu(n) - -(-n-2)u(n+2) = 0. \tag{HACT.2.1.2.2}$$

Initial conditions of HACT.2.1.2.2 are given by

$$\begin{aligned} u(0) &= -\frac{i}{2}\pi, \\ u(1) &= 1. \end{aligned} \tag{HACT.2.1.2.3}$$

HACT.2.2 Asymptotic expansion at 1.

HACT.2.2.1 First terms.

$$\operatorname{arccoth}(x) \approx \left(\frac{\ln(2)}{2} - \frac{x}{4} + \frac{1}{4} + \frac{(x-1)^2}{16} - \frac{(x-1)^3}{48} + \frac{(x-1)^4}{128} - \frac{(x-1)^5}{320} + \right. \tag{HACT.2.2.1.1}$$
$$\left. \frac{(x-1)^6}{768} - \frac{(x-1)^7}{1792} + \frac{(x-1)^8}{4096} + \frac{\ln(x-1)}{2} \cdots \right).$$

HACT.2.2.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

HACT.2.3 Asymptotic expansion at -1.

HACT.2.3.1 First terms.

$$\operatorname{arccoth}(x) \approx \left(\frac{-\ln(2)}{2} - \frac{x}{4} - \frac{1}{4} - \frac{(x+1)^2}{16} - \frac{(x+1)^3}{48} - \frac{(x+1)^4}{128} - \frac{(x+1)^5}{320} \right. \tag{HACT.2.3.1.1}$$
$$\left. - \frac{(x+1)^6}{768} - \frac{(x+1)^7}{1792} - \frac{(x+1)^8}{4096} - \frac{\ln(x+1)}{2} \cdots \right).$$

HACT.2.3.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

HACT.3 Graphs

HACT.3.1 Real axis.

HACT.3.2 Complex plane.