ATN Inverse Tangent

ATN.1 Introduction

Let x be a complex variable of $\mathbb{C} \setminus \{i, -i\}$.The function Inverse Tangent (noted arctan) is defined by the following second order differential equation

$$-2x\frac{\partial y(x)}{\partial x} - -\left(-1 - x^2\right)\frac{\partial^2 y(x)}{\partial x^2} = 0. \tag{ATN.1.1}$$

The initial conditions of ATN.1.1 are given at 0 by

$$\begin{aligned} \arctan(0) &= 0, \\ \frac{\partial \arctan(x)}{\partial x}(0) &= 1. \end{aligned} \tag{ATN.1.2}$$

ATN.2 Series and asymptotic expansions

ATN.2.1 Asymptotic expansion at i.

ATN.2.1.1 First terms.

$$\begin{aligned} \arctan(x) \approx \Bigg(& \frac{i}{2}\ln(2) + \frac{\left(x - \mathrm{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)\mathrm{RootOf}_{\xi,1}\left(1+\xi^2\right)}{4} \\ & - \frac{\left(x - \mathrm{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^2}{16} + \frac{\left(x - \mathrm{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^3}{48\,\mathrm{RootOf}_{\xi,1}\left(1+\xi^2\right)} - \\ & \frac{\left(x - \mathrm{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^4}{128\,\mathrm{RootOf}_{\xi,1}\left(1+\xi^2\right)^2} + \frac{\left(x - \mathrm{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^5}{320\,\mathrm{RootOf}_{\xi,1}\left(1+\xi^2\right)^3} - \\ & \frac{\left(x - \mathrm{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^6}{768\,\mathrm{RootOf}_{\xi,1}\left(1+\xi^2\right)^4} + \frac{\left(x - \mathrm{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^7}{1792\,\mathrm{RootOf}_{\xi,1}\left(1+\xi^2\right)^5} - \\ & \frac{\left(x - \mathrm{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^8}{4096\,\mathrm{RootOf}_{\xi,1}\left(1+\xi^2\right)^6} + \frac{\ln\left(x - \mathrm{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)}{2} \dots \Bigg). \end{aligned} \tag{ATN.2.1.1.1}$$

ATN.2.1.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

ATN.2.2 Asymptotic expansion at $-i$.

ATN.2.2.1 First terms.

$$\arctan(x) \approx \Bigg(-\frac{i}{2}\ln(2) + \frac{\left(x-\mathrm{RootOf}_{\xi,2}(1+\xi^2)\right)\mathrm{RootOf}_{\xi,2}(1+\xi^2)}{4} - \frac{\left(x-\mathrm{RootOf}_{\xi,2}(1+\xi^2)\right)^2}{16} + \frac{\left(x-\mathrm{RootOf}_{\xi,2}(1+\xi^2)\right)^3}{48\,\mathrm{RootOf}_{\xi,2}(1+\xi^2)} - \frac{\left(x-\mathrm{RootOf}_{\xi,2}(1+\xi^2)\right)^4}{128\,\mathrm{RootOf}_{\xi,2}(1+\xi^2)^2} + \frac{\left(x-\mathrm{RootOf}_{\xi,2}(1+\xi^2)\right)^5}{320\,\mathrm{RootOf}_{\xi,2}(1+\xi^2)^3} - \frac{\left(x-\mathrm{RootOf}_{\xi,2}(1+\xi^2)\right)^6}{768\,\mathrm{RootOf}_{\xi,2}(1+\xi^2)^4} + \frac{\left(x-\mathrm{RootOf}_{\xi,2}(1+\xi^2)\right)^7}{1792\,\mathrm{RootOf}_{\xi,2}(1+\xi^2)^5} - \frac{\left(x-\mathrm{RootOf}_{\xi,2}(1+\xi^2)\right)^8}{4096\,\mathrm{RootOf}_{\xi,2}(1+\xi^2)^6} + \frac{\ln\left(x-\mathrm{RootOf}_{\xi,2}(1+\xi^2)\right)}{2}\cdots\Bigg). \tag{ATN.2.2.1.1}$$

ATN.2.2.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

ATN.2.3 Taylor expansion at 0**.**

ATN.2.3.1 First terms.

$$\arctan(x) = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \frac{1}{9}x^9 - \frac{1}{11}x^{11} + \frac{1}{13}x^{13} - \frac{1}{15}x^{15} + \mathrm{O}(x^{16}). \tag{ATN.2.3.1.1}$$

ATN.2.3.2 General form.

$$\arctan(x) = \sum_{n=0}^{\infty} u(n)x^n. \tag{ATN.2.3.2.1}$$

The coefficients $u(n)$ satisfy the recurrence

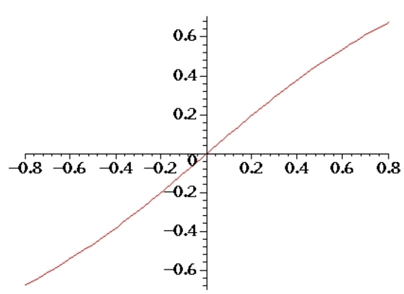
$$-nu(n) - -(-n-2)u(n+2) = 0. \tag{ATN.2.3.2.2}$$

Initial conditions of ATN.2.3.2.2 are given by

$$u(0) = 0, \quad u(1) = 1. \tag{ATN.2.3.2.3}$$

ATN.3 Graphs

ATN.3.1 Real axis.



ATN.3.2 Complex plane.