ASN Inverse Sine

ASN.1 Introduction

Let x be a complex variable of $\mathbb{C}\setminus\{\infty\}$.The function Inverse Sine (noted arcsin) is defined by the following second order differential equation

$$x\frac{\partial y(x)}{\partial x} + \left(x^2-1\right)\frac{\partial^2 y(x)}{\partial x^2} = 0. \tag{ASN.1.1}$$

The initial conditions of ASN.1.1 are given at 0 by

$$\begin{aligned} \arcsin(0) &= 0,\\ \frac{\partial \arcsin(x)}{\partial x}(0) &= 1. \end{aligned} \tag{ASN.1.2}$$

Related functions: Inverse Cosine,Inverse Hyperbolic Cosine

ASN.2 Series and asymptotic expansions

ASN.2.1 Asymptotic expansion at -1.

ASN.2.1.1 First terms.

$$\begin{aligned} \arcsin(x) \approx \left(\frac{-\pi}{2}\dots\right) + \sqrt{x+1}\Bigg(\sqrt{2} + \frac{(x+1)\sqrt{2}}{12} + \frac{3(x+1)^2\sqrt{2}}{160} + \\ \frac{5(x+1)^3\sqrt{2}}{896} + \frac{35(x+1)^4\sqrt{2}}{18432} + \frac{63(x+1)^5\sqrt{2}}{90112} + \\ \frac{231(x+1)^6\sqrt{2}}{851968} + \frac{143(x+1)^7\sqrt{2}}{1310720} + \frac{6435(x+1)^8\sqrt{2}}{142606336}\dots\Bigg). \end{aligned} \tag{ASN.2.1.1.1}$$

ASN.2.1.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

ASN.2.2 Asymptotic expansion at ∞.

ASN.2.2.1 First terms.

$$\arcsin(x) \approx \left(-i\ln(2i) - \frac{i}{4x^2} - \frac{3i}{32x^4} - \frac{5i}{96x^6} - \frac{35i}{1024x^8} - i\ln\left(\frac{1}{x}\right)\dots\right). \tag{ASN.2.2.1.1}$$

ASN.2.2.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

ASN.2.3 Taylor expansion at 0.

ASN.2.3.1 First terms.

$$\arcsin(x) = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \frac{5}{112}x^7 + \frac{35}{1152}x^9 + \frac{63}{2816}x^{11} + \frac{231}{13312} x^{13} + \frac{143}{10240}x^{15} + \mathrm{O}\left(x^{16}\right). \tag{ASN.2.3.1.1}$$

ASN.2.3.2 General form.

$$\arcsin(x) = \sum_{n=0}^{\infty} u(n)x^n. \tag{ASN.2.3.2.1}$$

The coefficients $u(n)$ satisfy the recurrence

$$n^2u(n) - -\left(-n^2 - 3n - 2\right)u(n+2) = 0. \tag{ASN.2.3.2.2}$$

Initial conditions of ASN.2.3.2.2 are given by

$$u(1) = 1, \quad u(0) = 0. \tag{ASN.2.3.2.3}$$

ASN.2.4 Asymptotic expansion at 1.

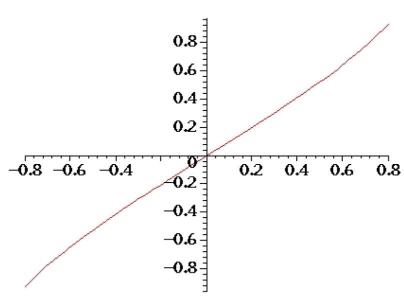
ASN.2.4.1 First terms.

$$\arcsin(x) \approx \left(\frac{\pi}{2}\dots\right) + \sqrt{x-1}\Big(-i\sqrt{2} + \frac{i}{12}(x-1)\sqrt{2} - \frac{3i}{160}\sqrt{2}(x-1)^2 + \frac{5i}{896}(x-1)^3\sqrt{2} - \frac{35i}{18432}\sqrt{2}(x-1)^4 + \frac{63i}{90112}(x-1)^5\sqrt{2} - \frac{231i}{851968}\sqrt{2}(x-1)^6 + \frac{143i}{1310720}(x-1)^7\sqrt{2} - \frac{6435i}{142606336}\sqrt{2}(x-1)^8\dots\Big). \tag{ASN.2.4.1.1}$$

ASN.2.4.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

ASN.3 Graphs

ASN.3.1 Real axis.



ASN.3.2 Complex plane.