ASC Inverse Secant

ASC.1 Introduction

Let x be a complex variable of $\mathbb{C} \setminus \{0\}$.The function Inverse Secant (noted arcsec) is defined by the following second order differential equation

$$\left(2x^2 - 1\right)\frac{\partial y(x)}{\partial x} + \left(x^3 - x\right)\frac{\partial^2 y(x)}{\partial x^2} = 0. \tag{ASC.1.1}$$

The initial conditions of ASC.1.1 at 0 are not simple to state, since 0 is a (regular) singular point.

Related functions: Inverse Hyperbolic Secant,Inverse Cosecant

ASC.2 Series and asymptotic expansions

ASC.2.1 Asymptotic expansion at -1.

ASC.2.1.1 First terms.

$$\begin{aligned}\operatorname{arcsec}(x) \approx (\pi \ldots) + \sqrt{x+1}\Bigg(&-i\sqrt{2} - \frac{5i}{12}(x+1)\sqrt{2} - \frac{43i}{160}(x+1)^2\sqrt{2} \\ &- \frac{177i}{896}(x+1)^3\sqrt{2} - \frac{2867i}{18432}(x+1)^4\sqrt{2} - \\ &\frac{11531i}{90112}(x+1)^5\sqrt{2} - \frac{92479i}{851968}(x+1)^6\sqrt{2} - \\ &\frac{74069i}{786432}(x+1)^7\sqrt{2} - \frac{11857475i}{142606336}(x+1)^8\sqrt{2}\ldots\Bigg).\end{aligned} \tag{ASC.2.1.1.1}$$

ASC.2.1.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

ASC.2.2 Asymptotic expansion at 0.

ASC.2.2.1 First terms.

$$\begin{aligned}&\operatorname{arcsec}(x) \approx \\ &\left(i\ln(2) + \frac{i}{4}x^2 + \frac{3i}{32}x^4 + \frac{5i}{96}x^6 + \frac{35i}{1024}x^8 + i\ln(x)\ldots\right).\end{aligned} \tag{ASC.2.2.1.1}$$

ASC.2.2.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

ASC.2.3 Asymptotic expansion at 1.

ASC.2.3.1 First terms.

$$\operatorname{arcsec}(x) \approx \sqrt{x-1}\left(\sqrt{2} - \frac{5\sqrt{2}(x-1)}{12} + \frac{43\sqrt{2}(x-1)^2}{160} - \frac{177\sqrt{2}(x-1)^3}{896} + \frac{2867\sqrt{2}(x-1)^4}{18432} - \frac{11531\sqrt{2}(x-1)^5}{90112} + \frac{92479\sqrt{2}(x-1)^6}{851968} - \frac{74069\sqrt{2}(x-1)^7}{786432} + \frac{11857475\sqrt{2}(x-1)^8}{142606336} - \frac{47442055\sqrt{2}(x-1)^9}{637534208} + \frac{126527543\sqrt{2}(x-1)^{10}}{1879048192} - \frac{1518418695\sqrt{2}(x-1)^{11}}{24696061952} + \frac{24295375159\sqrt{2}(x-1)^{12}}{429496729600} - \frac{97182800711\sqrt{2}(x-1)^{13}}{1855425871872} + \frac{777467420263\sqrt{2}(x-1)^{14}}{15942918602752} - \frac{3109879375897\sqrt{2}(x-1)^{15}}{68169720922112} \dots\right). \tag{ASC.2.3.1.1}$$

ASC.2.3.2 General form.

$$\operatorname{arcsec}(x) \approx \sqrt{x-1}\sum_{n=0}^{\infty} u(n)(x-1)^n. \tag{ASC.2.3.2.1}$$

The coefficients $u(n)$ satisfy the recurrence

$$2u(n)\left(\frac{1}{2}+n\right)n + u(n-1)\left(-\frac{1}{2}+n\right)\left(-\frac{1}{2}+3n\right) + u(n-2)\left(-\frac{3}{2}+n\right)\left(-\frac{1}{2}+n\right) = 0. \tag{ASC.2.3.2.2}$$

Initial conditions of ASC.2.3.2.2 are given by

$$u(0) = \sqrt{2}, \quad u(1) = \frac{-5\sqrt{2}}{12}. \tag{ASC.2.3.2.3}$$