ACT Inverse Cotangent

ACT.1 Introduction

Let x be a complex variable of $\mathbb{C}\setminus\{i,-i\}$.The function Inverse Cotangent (noted arccot) is defined by the following second order differential equation

$$2x\frac{\partial y(x)}{\partial x}+\left(1+x^2\right)\frac{\partial^2 y(x)}{\partial x^2}=0. \tag{ACT.1.1}$$

The initial conditions of ACT.1.1 are given at 0 by

$$\begin{aligned}\operatorname{arccot}(0)&=\frac{\pi}{2},\\ \frac{\partial \operatorname{arccot}(x)}{\partial x}(0)&=-1.\end{aligned} \tag{ACT.1.2}$$

ACT.2 Series and asymptotic expansions

ACT.2.1 Asymptotic expansion at $-i$.

ACT.2.1.1 First terms.

$$\begin{aligned}\operatorname{arccot}(x)\approx\Bigg(&\frac{\pi}{2}+\frac{i}{2}\ln(2)+\\ &\frac{i}{4}\operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)\left(x-\operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)\right)-\\ &\frac{i}{16}\left(x-\operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)\right)^2+\frac{i\left(x-\operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)\right)^3}{48\operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)}-\\ &\frac{i\left(x-\operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)\right)^4}{128\operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)^2}+\frac{i\left(x-\operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)\right)^5}{320\operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)^3}-\\ &\frac{i\left(x-\operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)\right)^6}{768\operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)^4}+\frac{i\left(x-\operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)\right)^7}{1792\operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)^5}-\\ &\frac{i\left(x-\operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)\right)^8}{4096\operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)^6}+\frac{i}{2}\ln\left(x-\operatorname{RootOf}_{\xi,2}\left(1+\xi^2\right)\right)\ldots\Bigg).\end{aligned} \tag{ACT.2.1.1.1}$$

ACT.2.1.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

ACT.2.2 Asymptotic expansion at i.

ACT.2.2.1 First terms.

$$\begin{aligned}\operatorname{arccot}(x) \approx \Bigg(&\frac{\pi}{2} - \frac{i}{2}\ln(2) - \\ &\frac{i}{4}\operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)\left(x - \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)\right) + \\ &\frac{i}{16}\left(x - \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^2 - \frac{i\left(x - \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^3}{48\operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)} + \\ &\frac{i\left(x - \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^4}{128\operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)^2} - \frac{i\left(x - \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^5}{320\operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)^3} + \\ &\frac{i\left(x - \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^6}{768\operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)^4} - \frac{i\left(x - \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^7}{1792\operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)^5} + \\ &\frac{i\left(x - \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)^8}{4096\operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)^6} - \frac{i}{2}\ln\left(x - \operatorname{RootOf}_{\xi,1}\left(1+\xi^2\right)\right)\ldots\Bigg).\end{aligned} \tag{ACT.2.2.1.1}$$

ACT.2.2.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

ACT.2.3 Taylor expansion at 0.

ACT.2.3.1 First terms.

$$\begin{aligned}\operatorname{arccot}(x) = &\frac{\pi}{2} - x + \frac{1}{3}x^3 - \frac{1}{5}x^5 + \frac{1}{7}x^7 - \frac{1}{9}x^9 + \frac{1}{11}x^{11} - \frac{1}{13}x^{13} + \frac{1}{15}x^{15} \\ &+ \mathrm{O}\left(x^{16}\right).\end{aligned} \tag{ACT.2.3.1.1}$$

ACT.2.3.2 General form.

$$\operatorname{arccot}(x) = \sum_{n=0}^{\infty} u(n)x^n. \tag{ACT.2.3.2.1}$$

The coefficients $u(n)$ satisfy the recurrence

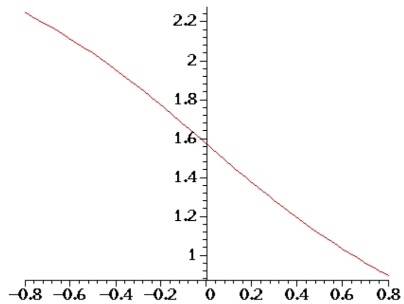
$$nu(n) + (n+2)u(n+2) = 0. \tag{ACT.2.3.2.2}$$

Initial conditions of ACT.2.3.2.2 are given by

$$\begin{aligned}u(0) &= \frac{\pi}{2}, \\ u(1) &= -1.\end{aligned} \tag{ACT.2.3.2.3}$$

ACT.3 Graphs

ACT.3.1 Real axis.



ACT.3.2 Complex plane.