ACS Inverse Cosine

ACS.1 Introduction

Let x be a complex variable of $\mathbb{C} \setminus \{\infty\}$.The function Inverse Cosine (noted arccos) is defined by the following second order differential equation

$$x\frac{\partial y(x)}{\partial x} + \left(x^2 - 1\right)\frac{\partial^2 y(x)}{\partial x^2} = 0. \tag{ACS.1.1}$$

The initial conditions of ACS.1.1 are given at 0 by

$$\begin{aligned} \arccos(0) &= \frac{\pi}{2}, \\ \frac{\partial \arccos(x)}{\partial x}(0) &= -1. \end{aligned} \tag{ACS.1.2}$$

Related functions: Inverse Sine,Inverse Hyperbolic Cosine

ACS.2 Series and asymptotic expansions

ACS.2.1 Asymptotic expansion at -1.

ACS.2.1.1 First terms.

$$\begin{aligned} \arccos(x) \approx (\pi \ldots) + \sqrt{x+1}\Bigg(-\sqrt{2} - \frac{(x+1)\sqrt{2}}{12} - \frac{3(x+1)^2\sqrt{2}}{160} - \\ \frac{5(x+1)^3\sqrt{2}}{896} - \frac{35(x+1)^4\sqrt{2}}{18432} - \frac{63(x+1)^5\sqrt{2}}{90112} - \\ \frac{231(x+1)^6\sqrt{2}}{851968} - \frac{143(x+1)^7\sqrt{2}}{1310720} - \frac{6435(x+1)^8\sqrt{2}}{142606336} \ldots\Bigg). \end{aligned} \tag{ACS.2.1.1.1}$$

ACS.2.1.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

ACS.2.2 Asymptotic expansion at 1.

ACS.2.2.1 First terms.

(ACS.2.2.1.1)
$$\arccos(x) \approx \sqrt{x-1}\Big(-\frac{5i}{896}\sqrt{2}(x-1)^3 + \frac{231i}{851968}\sqrt{2}(x-1)^6 - \frac{9694845i}{68169720922112}\sqrt{2}(x-1)^{15} + i\sqrt{2} + \frac{46189i}{5637144576}\sqrt{2}(x-1)^{10} - \frac{1300075i}{1855425871872}\sqrt{2}(x-1)^{13} + \frac{6435i}{142606336}\sqrt{2}(x-1)^8 - \frac{143i}{1310720}\sqrt{2}(x-1)^7 + \frac{35i}{18432}\sqrt{2}(x-1)^4 + \frac{5014575i}{15942918602752}\sqrt{2}(x-1)^{14} - \frac{88179i}{24696061952}\sqrt{2}(x-1)^{11} - \frac{i}{12}\sqrt{2}(x-1) + \frac{3i}{160}\sqrt{2}(x-1)^2 - \frac{63i}{90112}\sqrt{2}(x-1)^5 - \frac{12155i}{637534208}\sqrt{2}(x-1)^9 + \frac{676039i}{429496729600}\sqrt{2}(x-1)^{12}\dots\Big).$$

ACS.2.2.2 General form.

(ACS.2.2.2.1)
$$\arccos(x) \approx \sqrt{x-1}\sum_{n=0}^{\infty} u(n)(x-1)^n.$$

The coefficients $u(n)$ satisfy the recurrence

(ACS.2.2.2.2)
$$2u(n)\left(n+\frac{1}{2}\right)n + u(n-1)\left(-\frac{1}{2}+n\right)^2 = 0.$$

Initial conditions of ACS.2.2.2.2 are given by

(ACS.2.2.2.3)
$$u(0) = i\sqrt{2}.$$

The recurrence ACS.2.2.2.2 has the closed form solution

(ACS.2.2.2.4)
$$u(n) = \frac{i2^{\left(n+\frac{1}{2}\right)}\Gamma\left(n+\frac{1}{2}\right)(-1)^n}{4^n\Gamma(n+1)\sqrt{\pi}(2n+1)}.$$

ACS.2.3 Asymptotic expansion at ∞.

ACS.2.3.1 First terms.

(ACS.2.3.1.1)
$$\arccos(x) \approx \left(i\ln(2) + \frac{i}{4x^2} + \frac{3i}{32x^4} + \frac{5i}{96x^6} + \frac{35i}{1024x^8} + i\ln\left(\frac{1}{x}\right)\dots\right).$$

ACS.2.3.2 General form. The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

ACS.2.4 Taylor expansion at 0.

ACS.2.4.1 First terms.

(ACS.2.4.1.1)
$$\arccos(x) = \frac{\pi}{2} - x - \frac{1}{6}x^3 - \frac{3}{40}x^5 - \frac{5}{112}x^7 - \frac{35}{1152}x^9 - \frac{63}{2816}x^{11} - \frac{231}{13312}x^{13} - \frac{143}{10240}x^{15} + \mathrm{O}\left(x^{16}\right).$$

ACS.2.4.2 General form.

(ACS.2.4.2.1) $$\arccos(x) = \sum_{n=0}^{\infty} u(n)x^n.$$

The coefficients $u(n)$ satisfy the recurrence

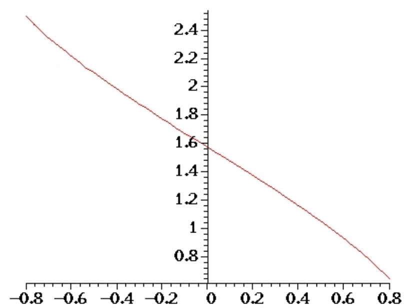
(ACS.2.4.2.2) $$n^2u(n) - -\left(-n^2 - 3n - 2\right)u(n+2) = 0.$$

Initial conditions of ACS.2.4.2.2 are given by

(ACS.2.4.2.3) $$u(0) = \frac{\pi}{2}, \quad u(1) = -1.$$

ACS.3 Graphs

ACS.3.1 Real axis.



ACS.3.2 Complex plane.