

# ACCS Inverse Cosecant

## ACCS.1 Introduction

Let  $x$  be a complex variable of  $\mathbb{C} \setminus \{0\}$ . The function Inverse Cosecant (noted arccsc) is defined by the following second order differential equation

$$(ACCS.1.1) \quad (2x^2 - 1) \frac{\partial y(x)}{\partial x} + (x^3 - x) \frac{\partial^2 y(x)}{\partial x^2} = 0.$$

The initial conditions of ACCS.1.1 at 0 are not simple to state, since 0 is a (regular) singular point.

Related functions: Inverse Hyperbolic Secant, Inverse Secant

## ACCS.2 Series and asymptotic expansions

### ACCS.2.1 Asymptotic expansion at $-1$ .

*ACCS.2.1.1 First terms.*

$$(ACCS.2.1.1.1) \quad \begin{aligned} \operatorname{arccsc}(x) \approx & \left( \frac{-\pi}{2} \dots \right) + \sqrt{x+1} \left( i\sqrt{2} + \frac{5i}{12}(x+1)\sqrt{2} + \right. \\ & \frac{43i}{160}(x+1)^2\sqrt{2} + \frac{177i}{896}(x+1)^3\sqrt{2} + \frac{2867i}{18432}(x+1)^4\sqrt{2} + \\ & \frac{11531i}{90112}(x+1)^5\sqrt{2} + \frac{92479i}{851968}(x+1)^6\sqrt{2} + \\ & \left. \frac{74069i}{786432}(x+1)^7\sqrt{2} + \frac{11857475i}{142606336}(x+1)^8\sqrt{2} \dots \right). \end{aligned}$$

*ACCS.2.1.2 General form.* The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

### ACCS.2.2 Asymptotic expansion at 0.

*ACCS.2.2.1 First terms.*

$$(ACCS.2.2.1.1) \quad \begin{aligned} \operatorname{arccsc}(x) \approx & \left( \right. \\ & \left. -i \ln(2) + \frac{\pi}{2} - \frac{i}{4}x^2 - \frac{3i}{32}x^4 - \frac{5i}{96}x^6 - \frac{35i}{1024}x^8 - i \ln(x) \dots \right). \end{aligned}$$

*ACCS.2.2.2 General form.* The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).

### ACCS.2.3 Asymptotic expansion at 1.

*ACCS.2.3.1 First terms.*

$$\begin{aligned} \operatorname{arccsc}(x) \approx & \left( \frac{\pi}{2} \dots \right) + \sqrt{x-1} \left( -\sqrt{2} + \frac{5(x-1)\sqrt{2}}{12} - \frac{43(x-1)^2\sqrt{2}}{160} + \right. \\ & \frac{177(x-1)^3\sqrt{2}}{896} - \frac{2867(x-1)^4\sqrt{2}}{18432} + \frac{11531(x-1)^5\sqrt{2}}{90112} - \\ (ACCS.2.3.1.1) & \frac{92479(x-1)^6\sqrt{2}}{851968} + \frac{74069(x-1)^7\sqrt{2}}{786432} - \\ & \left. \frac{11857475(x-1)^8\sqrt{2}}{142606336} \dots \right). \end{aligned}$$

*ACCS.2.3.2 General form.* The general form of is not easy to state and requires to exhibit the basis of formal solutions of ?? (coming soon).