The file table3.pdf is associated with the paper:

"A Class of 1-Additive Sequences and Quadratic Recurrences",
Julien Cassaigne and Steven R. Finch,
Experimental Mathematics, vol. 4 (1995) no. 1, pp. 49-60,

and constitutes the complete seven-page version of Table 3, which could not appear in the paper due to its length.

As in the paper, Table 3 indicates an admissible value of d for each of 55 possible pairs (X,Y). The remaining 45 pairs are obtained via the symmetry (p,q,d) ---> (q,p,-d). If there are several lines without conditions for a given case, then each of them is a solution. If there are several lines with conditions, then at least one of the conditions is true for each relevant value of (p,q). Following the value of d is the indication of the new sets, X' and Y', for which p+d is in X' and q-d is in Y', and of the corresponding new values for ℓ and h.

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(X, Y)	d	(X', Y')	(ℓ_{p+d}, ℓ_{q-d})	(h_{p+d}, h_{q-d})	condition
(A, A)	$w\,\ell_q - 4h_p$	(E, A)	$(\ell_p+\ell_q, 0)$	$(\_, h_p+h_q)$	$h_p+h_q \le 2k$
	$w\,\ell_q - 4h_p + 8k + 2$	(I, C)	$(\ell_p+\ell_q, 0)$	$(\_, h_p+h_q-2k)$	$h_p+h_q > 2k$ and $\ell_p+\ell_q$ odd
	$w\,\ell_q - 4\,h_p - 2$	(J, D)	$(\ell_p+\ell_q-1, 0)$	$(\_, h_p+h_q-2k)$	$h_p+h_q > 2k$ and $\ell_p+\ell_q$ even
(A, B)	$4k + 1$	(B, A)	(ℓ_p, ℓ_q)	(h_p, h_q)	$\ell_p \ne \ell_q$
	$4h_q - 4h_p$	(A, B)	(ℓ_p, ℓ_q)	(h_q, h_p)	$\ell_p = \ell_q$ and $h_p \ne h_q$
	w	(A, B)	(ℓ_p+1, ℓ_p-1)	(h_p, h_p)	$\ell_p=\ell_q$, $h_p=h_q$, $h^0+h^i=0$, $\ell^0+\ell^i=1$
	-4	(E, B)	(ℓ_p, ℓ_p)	$(\_, 2)$	$\ell_p=\ell_q$, $h_p=h_q=1$, $h^0+h^i=1$, $\ell^0+\ell^i=0$
	-4	(A, B)	(ℓ_p, ℓ_p)	(h_p-1, h_p+1)	$\ell_p=\ell_q$, $1<h_p=h_q<2k$, $h^0+h^i=1$, $\ell^0+\ell^i=0$
	-4	(A, H)	(ℓ_p, ℓ_p)	$(2k-1, \_)$	$\ell_p=\ell_q$, $h_p=h_q=2k$, $h^0+h^i=1$, $\ell^0+\ell^i=0$
	w - 4	(E, B)	(ℓ_p+1, ℓ_p-1)	$(\_, 2)$	$\ell_p=\ell_q$, $h_p=h_q=1$, $h^0+h^i=1$, $\ell^0+\ell^i=1$ (NB: $\ell_p>0$)
	w - 4	(A, B)	(ℓ_p+1, ℓ_p-1)	(h_p-1, h_p+1)	$\ell_p=\ell_q$, $1<h_p=h_q<2k$, $h^0+h^i=1$, $\ell^0+\ell^i=1$ (NB: $\ell_p>0$)
	w - 4	(A, H)	(ℓ_p+1, ℓ_p-1)	$(2k-1, \_)$	$\ell_p=\ell_q$, $h_p=h_q=2k$, $h^0+h^i=1$, $\ell^0+\ell^i=1$ (NB: $\ell_p>0$)
(A, C)	-2	(C, A)	(ℓ_p, ℓ_q)	(h_p, h_q)	$\ell_p \ne \ell_q$
	$4h_q - 4h_p$	(A, C)	(ℓ_p, ℓ_q)	(h_q, h_p)	$\ell_p = \ell_q$ and $h_p \ne h_q$
	$w\,\ell_q - 4h_p$	(E, C)	$(\ell_p+\ell_q, 0)$	$(\_, h_p+h_q)$	$h_p+h_q \le 2k$
	$w\,\ell_q - 4h_p$	(E, D)	$(\ell_p+\ell_q, 0)$	$(\_, h_p+h_q-2k-1)$	$h_p+h_q > 2k+1$

Table 3-1: Admissible value of d for each possible (X, Y)

(X, Y)	d	(X', Y')	(ℓ_{p+d}, ℓ_{q-d})	(h_{p+d}, h_{q-d})	condition
(A, D)	8k + 2	(D, A)	(ℓ_p, ℓ_q)	(h_p, h_q)	$\ell_p \neq \ell_q$
	$4h_q - 4h_p$	(A, D)	(ℓ_p, ℓ_q)	(h_q, h_p)	$\ell_p = \ell_q$ and $h_p \neq h_q$
	$w\ell_q - 4h_p$	(E, D)	$(\ell_p+\ell_q, 0)$	$(\_, h_p+h_q)$	$h_p+h_q \leq 2k$
	$w\ell_q - 4h_p + w$	(E, C)	$(\ell_p+\ell_q+1, 0)$	$(\_, h_p+h_q-2k-1)$	$h_p+h_q > 2k+1$
(A, E)	$-w\ell_p$	(A, E)	$(0, \ell_p+\ell_q)$	$(h_p, \_)$	$\ell_p > 0$
	-2	(C, C)	$(0, \ell_q)$	$(h_p, 1)$	$\ell_p = 0$ and ℓ_q even (NB: $\ell_q>0$ or $h_p>1$)
	8k + 2	(D, D)	$(0, \ell_q-1)$	$(h_p, 1)$	$\ell_p = 0$ and $\ell_q > 1$ odd
	4k + 1	(B, G)	$(\ell_p, 0)$	$(h_p, \_)$	$\ell_q = 1$
(A, F)	4k + 1	(B, E)	(ℓ_p, ℓ_q)	$(h_p, \_)$	
(A, G)	4k + 1	(B, D)	(ℓ_p, ℓ_q)	$(h_p, 1)$	
	8k + 2	(D, B)	(ℓ_p, ℓ_q)	$(h_p, 1)$	
(A, H)	-2	(C, G)	(ℓ_p, ℓ_q)	$(h_p, \_)$	
(A, I)	4k + 1	(B, F)	(ℓ_p, ℓ_q)	$(h_p, \_)$	
	8k + 2	(D, E)	(ℓ_p, ℓ_q)	$(h_p, \_)$	
(A, J)	-2	(C, E)	(ℓ_p, ℓ_q+1)	$(h_p, \_)$	
(B, B)	4k + 1	(D, A)	(ℓ_p, ℓ_q)	(h_p, h_q)	

Table 3-2

(X, Y)	d	(X', Y')	(ℓ_{p+d}, ℓ_{q-d})	(h_{p+d}, h_{q-d})	condition
(B, C)	-4k - 3	(C, B)	(ℓ_p, ℓ_q)	(h_p, h_q)	$\ell_p \neq \ell_q$
	$4h_q - 4h_p$	(B, C)	(ℓ_p, ℓ_q)	(h_q, h_p)	$\ell_p = \ell_q$ and $h_p \neq h_q$
	$w\,\ell_q - 4h_p$	(F, C)	$(\ell_p+\ell_q, 0)$	$(\_, h_p+h_q)$	$h_p+h_q \leq 2k$
	$w\,\ell_q - 4h_p$	(F, D)	$(\ell_p+\ell_q, 0)$	$(\_, h_p+h_q-2k-1)$	$h_p+h_q > 2k+1$
(B, D)	4k + 1	(D, B)	(ℓ_p, ℓ_q)	(h_p, h_q)	$\ell_p \neq \ell_q$
	$4h_q - 4h_p$	(B, D)	(ℓ_p, ℓ_q)	(h_q, h_p)	$\ell_p = \ell_q$ and $h_p \neq h_q$
	$w\,\ell_q - 4h_p$	(F, D)	$(\ell_p+\ell_q, 0)$	$(\_, h_p+h_q)$	$h_p+h_q \leq 2k$
	$w\,\ell_q - 4h_p + w$	(F, C)	$(\ell_p+\ell_q+1, 0)$	$(\_, h_p+h_q-2k-1)$	$h_p+h_q > 2k+1$
(B, E)	-4k - 1	(A, F)	(ℓ_p, ℓ_q)	$(h_p, \_)$	
(B, F)	4k + 1	(D, E)	(ℓ_p, ℓ_q)	$(h_p, \_)$	
(B, G)	-4k - 1	(A, E)	(ℓ_p, ℓ_q+1)	$(h_p, \_)$	
(B, H)	-4k - 3	(C, E)	(ℓ_p, ℓ_q+1)	$(h_p, \_)$	
(B, I)	4k + 1	(D, F)	(ℓ_p, ℓ_q)	$(h_p, \_)$	
(B, J)	-4k - 3	(C, F)	(ℓ_p, ℓ_q+1)	$(h_p, \_)$	
(C, C)	$w\,\ell_q - 4h_p + 2$	(E, A)	$(\ell_p+\ell_q, 0)$	$(\_, h_p+h_q-1)$	$h_p+h_q \leq 2k+1$
	$w\,\ell_q - 4h_p + 8k + 4$	(I, C)	$(\ell_p+\ell_q, 0)$	$(\_, h_p+h_q-2k-1)$	$h_p+h_q > 2k+1$ and $\ell_p+\ell_q$ odd
	$w\,\ell_q - 4h_p$	(J, D)	$(\ell_p+\ell_q-1, 0)$	$(\_, h_p+h_q-2k-1)$	$h_p+h_q > 2k+1$ and $\ell_p+\ell_q$ even

Table 3-3

(X, Y)	d	(X', Y')	(ℓ_{p+d}, ℓ_{q-d})	(h_{p+d}, h_{q-d})	condition
(C, D)	8k + 4	(D, C)	(ℓ_p, ℓ_q)	(h_p, h_q)	$\ell_p \neq \ell_q$
	$4h_q - 4h_p$	(C, D)	(ℓ_p, ℓ_q)	(h_q, h_p)	$\ell_p = \ell_q$ and $h_p \neq h_q$
	$w\,\ell_q - 4h_p$	(J, D)	$(\ell_p+\ell_q-1, 0)$	$(\_, h_p+h_q)$	$h_p+h_q \leq 2k$ and $\ell_p+\ell_q$ even
	$w\,\ell_q - 4h_p + w + 2$	(E, E)	$(\ell_p+\ell_q+1, 0)$		$h_p+h_q = 2k+2$
	$w\,\ell_q - 4h_p + w + 2$	(E, A)	$(\ell_p+\ell_q+1, 0)$	$(\_, h_p+h_q-2k-2)$	$h_p+h_q > 2k+2$
(C, E)	2	(A, J)	(ℓ_p, ℓ_q-1)	$(h_p, \_)$	$z(2k+1, \ell_q-1) = 0$
	4k + 3	(B, H)	(ℓ_p, ℓ_q-1)	$(h_p, \_)$	$z(2k+1, \ell_q-1) = 1$
(C, F)	4k + 3	(B, J)	(ℓ_p, ℓ_q-1)	$(h_p, \_)$	$z(2k+1, \ell_q-1) = 0$
	8k + 4	(D, H)	(ℓ_p, ℓ_q-1)	$(h_p, \_)$	$z(2k+1, \ell_q-1) = 1$
(C, G)	2	(A, H)	(ℓ_p, ℓ_q)	$(h_p, \_)$	
(C, H)	8k + 4	(D, F)	(ℓ_p, ℓ_q)	$(h_p, \_)$	
(C, I)	2	(A, A)	(ℓ_p, ℓ_q)	$(h_p, 2k)$	$z(2k+1, \ell_q-1) = 1$ and $\ell_p \neq \ell_q$
	8k + 4	(D, J)	(ℓ_p, ℓ_q-1)	$(h_p, \_)$	$z(2k+1, \ell_q-1) = 0$
	-8k - 4	(D, J)	(ℓ_p-1, ℓ_q)	$(h_p, \_)$	ℓ_p odd (NB: true if $z(2k+1, \ell_q-1) = 1$ and $\ell_p = \ell_q$)
(C, J)	$-w\,\ell_p - 2$	(A, E)	$(0, \ell_p+\ell_q+1)$	$(h_p-1, \_)$	$h_p > 1$
	$-w\,\ell_p - 2$	(E, E)	$(0, \ell_p+\ell_q+1)$		$h_p = 1$ (NB: $\ell_p+\ell_q+1 > 0$)

Table 3-4

(X, Y)	d	(X', Y')	(ℓ_{p+d}, ℓ_{q-d})	(h_{p+d}, h_{q-d})	condition
(D, D)	$w\,\ell_q - 4h_p + 8k + 6$	(E, A)	$(\ell_p+\ell_q+1, 0)$	$(\_, h_p+h_q-1)$	$h_p+h_q \le 2k+1$
	$w\,\ell_q - 4h_p + 8k + 4$	(J, D)	$(\ell_p+\ell_q, 0)$	$(\_, h_p+h_q-2k-1)$	$h_p+h_q > 2k+1$ and $\ell_p+\ell_q$ odd
	$w\,\ell_q - 4h_p + w$	(I, C)	$(\ell_p+\ell_q+1, 0)$	$(\_, h_p+h_q-2k-1)$	$h_p+h_q > 2k+1$ and $\ell_p+\ell_q$ even
(D, E)	$-4k - 1$	(B, F)	(ℓ_p, ℓ_q)	$(h_p, \_)$	
(D, F)	$-4k - 1$	(B, I)	(ℓ_p, ℓ_q)	$(h_p, \_)$	$z(2k+1, \ell_q) = 0$
	$-8k - 4$	(C, H)	(ℓ_p, ℓ_q)	$(h_p, \_)$	$z(2k+1, \ell_q) = 1$
(D, G)	$-4k - 1$	(B, E)	(ℓ_p, ℓ_q+1)	$(h_p, \_)$	
	$-8k - 2$	(A, F)	(ℓ_p, ℓ_q+1)	$(h_p, \_)$	
(D, H)	$-8k - 4$	(C, F)	(ℓ_p, ℓ_q+1)	$(h_p, \_)$	
(D, I)	$-8k - 4$	(C, J)	(ℓ_p, ℓ_q)	$(h_p, \_)$	
(D, J)	$-8k - 2$	(A, A)	(ℓ_p, ℓ_q+1)	$(h_p, 2k)$	$z(2k+1, \ell_q+1) = 1$ and $[\ell_p \ne \ell_q+1$ or $h_p \ne 2k]$
	$-8k - 4$	(C, I)	(ℓ_p, ℓ_q+1)	$(h_p, \_)$	$z(2k+1, \ell_q+1) = 0$
	$8k + 4$	(C, I)	(ℓ_p+1, ℓ_q)	$(h_p, \_)$	ℓ_p even and h_p even and $\ell_q \ge 0$
	-4	(D, C)	$(0, \ell_q+1)$	$(h_p-1, 1)$	$\ell_p=0$ and $h_p>1$ and ℓ_q odd
(E, E)	$-w\,\ell_q + 12k + 7$	(G, F)	$(0, \ell_p+\ell_q-1)$		(NB: $\ell_p+\ell_q > 0$)
(E, F)	w	(E, F)	(ℓ_p+1, ℓ_q-1)		$\ell_q > 0$
	$-w$	(E, F)	(ℓ_p-1, ℓ_q+1)		$\ell_p > 0$ (NB: $\ell_p>0$ or $\ell_q>0$)
(E, G)	$4k + 1$	(F, D)	(ℓ_p, ℓ_q)	$(\_, 1)$	

Table 3-5

(X, Y)	d	(X', Y')	(ℓ_{p+d}, ℓ_{q-d})	(h_{p+d}, h_{q-d})	condition
(E, H)	-16k - 4	(A, B)	$(\ell_p\text{-}1, \ell_q\text{+}1)$	(1, 2k)	ℓ_p is odd
	4	(A, B)	(ℓ_p, ℓ_q)	(1, 2k)	ℓ_p is even
(E, I)	$w\,\ell_q + 4k + 1$	(F, F)	$(\ell_p\text{+}\ell_q, 0)$		(NB: $\ell_p\text{+}\ell_q > 0$)
(E, J)	$w\,\ell_q + 4k + 1$	(F, H)	$(\ell_p\text{+}\ell_q, 0)$		(NB: $\ell_p\text{+}\ell_q \geq 0$)
(F, F)	$-w\,\ell_p + 20k + 9$	(I, E)	$(1, \ell_p\text{+}\ell_q\text{-}1)$		(NB: $\ell_p\text{+}\ell_q > 0$)
(F, G)	$-w\,\ell_p - 4k - 1$	(E, E)	$(0, \ell_p\text{+}\ell_q\text{+}1)$		
(F, H)	12k + 7	(E, J)	$(\ell_p\text{+}1, \ell_q\text{-}1)$		
(F, I)	$w\,\ell_q - w$	(F, I)	$(\ell_p\text{+}\ell_q\text{-}1, 1)$		$\ell_q > 1$
	-2w	(F, I)	$(\ell_p\text{-}2, \ell_q\text{+}2)$		$\ell_p > 1$ and $\ell_q = 1$
	-8k - 4	(H, J)	$(0, \ell_q)$		$\ell_p = 1$
	4	(B, C)	(0, 1)	(1, 2k)	$\ell_p = 0$ and $\ell_q = 1$
(F, J)	$w\,\ell_q + w$	(F, J)	$(\ell_p\text{+}\ell_q\text{+}1, -1)$		$\ell_p > -1$
	-2w	(F, J)	$(\ell_p\text{-}2, \ell_q\text{+}2)$		$\ell_p > 1$ and $\ell_q = -1$
	8k + 4	(H, I)	$(0, \ell_q)$		$\ell_p = 0$ (NB: true only if $\ell_q \geq 0$)
	-16k - 4	(B, D)	(0, 0)	(1, 2k)	$\ell_p = 1$ and $\ell_q = -1$
(G, G)	4k + 1	(E, D)	$(\ell_p\text{+}1, \ell_q)$	(_, 1)	
	8k + 2	(F, B)	$(\ell_p\text{+}1, \ell_q)$	(_, 1)	
	12k + 3	(I, A)	$(\ell_p\text{+}1, \ell_q)$	(_, 1)	

Table 3-6

(X, Y)	d	(X', Y')	(ℓ_{p+d}, ℓ_{q-d})	(h_{p+d}, h_{q-d})	condition
(G, H)	-12k - 5	(C, I)	(ℓ_p, ℓ_q+1)	(1, _)	
	12k + 3	(I, C)	(ℓ_p+1, ℓ_q)	(_, 1)	
(G, I)	4k + 1	(E, F)	(ℓ_p+1, ℓ_q)		
	8k + 2	(F, E)	(ℓ_p+1, ℓ_q)		
(G, J)	-2	(H, E)	(ℓ_p, ℓ_q+1)		
(H, H)	12k + 1	(C, A)	(ℓ_p+1, ℓ_q)	(2k, 1)	
	12k + 3	(A, C)	(ℓ_p+1, ℓ_q)	(2k, 1)	
	12k + 5	(I, E)	(ℓ_p+1, ℓ_q)		
(H, I)	-8k - 4	(F, J)	(ℓ_p, ℓ_q)		
(H, J)	8k + 4	(F, I)	(ℓ_p+1, ℓ_q)		$\ell_q > -1$
	-8k	(B, C)	$(\ell_p, 0)$	(1, 2k)	$\ell_q = -1$
(I, I)	$w\,\ell_q - 8k$	(C, D)	$(\ell_p+\ell_q, 0)$	(1, 2k)	$\ell_p+\ell_q$ even
	$w\,\ell_q - 8k - w$	(C, D)	$(\ell_p+\ell_q-1, 1)$	(1, 2k)	$\ell_p+\ell_q$ odd (NB: $\ell_p+\ell_q>0$)
(I, J)	$w\,\ell_q + 4$	(D, D)	$(\ell_p+\ell_q, 0)$	(1, 2k)	$\ell_p+\ell_q$ even
	$w\,\ell_q + 4 - w$	(D, D)	$(\ell_p+\ell_q-1, 1)$	(1, 2k)	$\ell_p+\ell_q$ odd (NB: $\ell_p+\ell_q>0$)
(J, J)	$w\,\ell_q + 4$	(C, D)	$(\ell_p+\ell_q+1, 0)$	(1, 2k)	$\ell_p+\ell_q$ odd
	$w\,\ell_q + 4 - w$	(C, D)	$(\ell_p+\ell_q, 1)$	(1, 2k)	$\ell_p+\ell_q$ even

Table 3-7